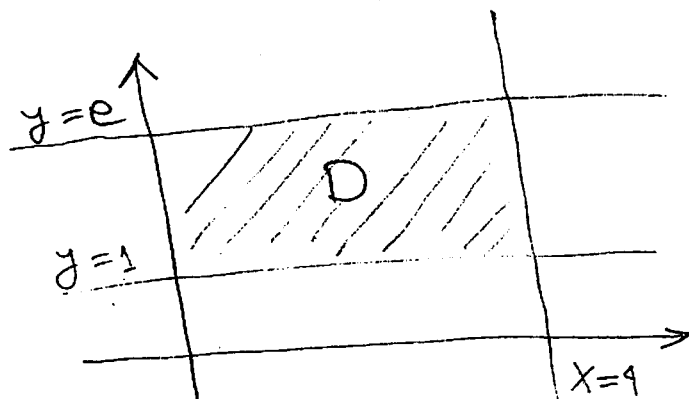


① $\iint_D x \ln y \, dx \, dy$, $D = \{(x, y) \mid 0 \leq x \leq 4 \wedge 1 \leq y \leq e\}$



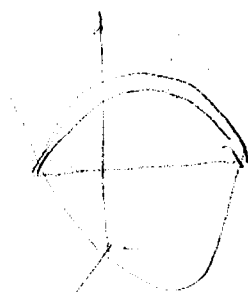
$$\int_0^4 x \, dx \int_1^e \ln y \, dy = \int_0^4 x \, dx \cdot \left\{ \begin{array}{l} u = \ln y \Rightarrow du = \frac{1}{y} dy \\ v = y \end{array} \right\}$$

$$= \int_0^4 x \, dx \cdot \left(y \ln y \Big|_1^e - \int_1^e dy \right) =$$

$$= \int_0^4 x \, dx \cdot (e \ln e - \ln 1) - (e - 1) =$$

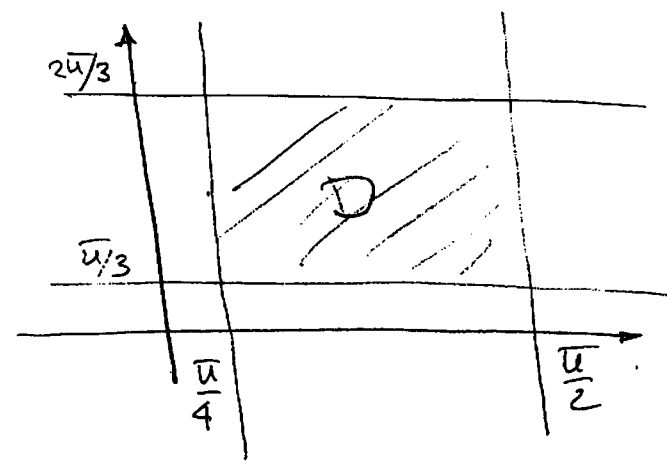
$$= \int_0^4 x \, dx \cdot (e - e + 1) = \int_0^4 x \, dx = \frac{x^2}{2} \Big|_0^4 =$$

$$= \frac{1}{2} (16 - 0) = \underline{\underline{8}}$$



3. $\iint_D (\cos^2 x + \sin^2 y) dx dy$

$$D = \left\{ (x, y) \mid \frac{\pi}{4} \leq x \leq \frac{\pi}{2} \wedge \frac{\pi}{3} \leq y \leq \frac{2\pi}{3} \right\}$$



$$= \int_{\pi/4}^{\pi/2} dx \int_{\pi/3}^{2\pi/3} (\cos^2 x + \sin^2 y) dy =$$

$$= \int_{\pi/4}^{\pi/2} dx \left[\int_{\pi/3}^{2\pi/3} \cos^2 x dy + \int_{\pi/3}^{2\pi/3} \sin^2 y dy \right] =$$

$$= \int_{\pi/4}^{\pi/2} dx \cdot \left(\cos^2 x y \Big|_{\pi/3}^{2\pi/3} + \left(-\frac{\sin y \cos y}{2} + \frac{1}{2} \int \sin y dy \right) \Big|_{\pi/3}^{2\pi/3} \right)$$

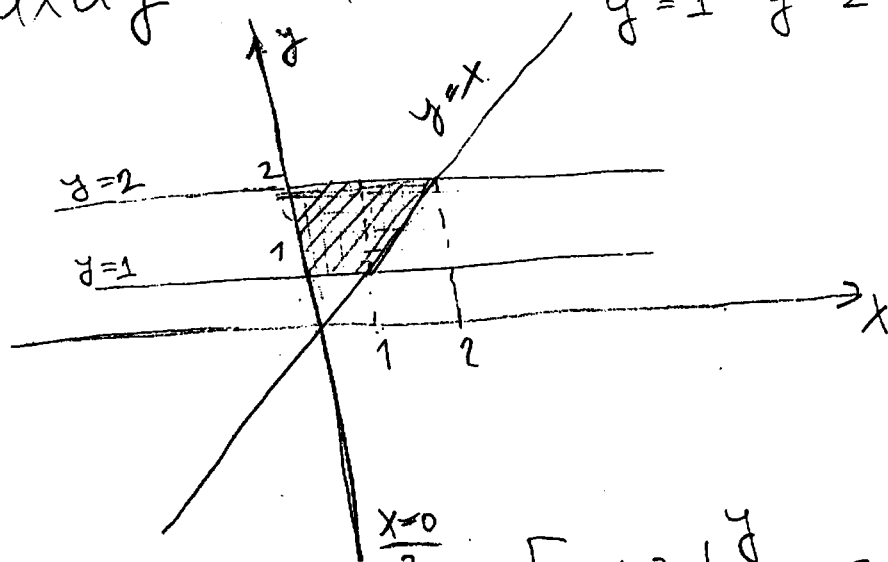
$$= \int_{\pi/4}^{\pi/2} dx \cdot \left[\cos^2 x \left(\frac{2\pi}{3} - \frac{\pi}{3} \right) - \frac{\sin 2y}{4} \Big|_{\pi/3}^{2\pi/3} + \frac{1}{2} \cos y \Big|_{\pi/3}^{2\pi/3} \right]$$

$$= \int_{\pi/4}^{\pi/2} dx \left[\cos^2 x \cdot \left(\frac{\pi}{3} \right) - \frac{1}{4} \left(\sin \frac{4\pi}{3} - \sin \frac{2\pi}{3} \right) - \frac{1}{2} \left(\cos \frac{2\pi}{3} - \cos \frac{\pi}{3} \right) \right]$$

$$= \int_{\pi/4}^{\pi/2} dx \left[\frac{\pi}{3} \cos^2 x - \frac{1}{4} \left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \right) - \frac{1}{2} \left(-\frac{1}{2} - \frac{1}{2} \right) \right] =$$

$$= \frac{\pi}{3} \int_{\pi/4}^{\pi/2} \cos^2 x \, dx + \frac{\sqrt{3}}{4} \int_{\pi/4}^{\pi/2} dx + \frac{1}{2} \int_{\pi/4}^{\pi/2} dx = \dots$$

⑤ $\iint_D (x^2 + y^2) \, dx \, dy = D$ ограничена $x=0, y=x$
 $y=1, y=2$

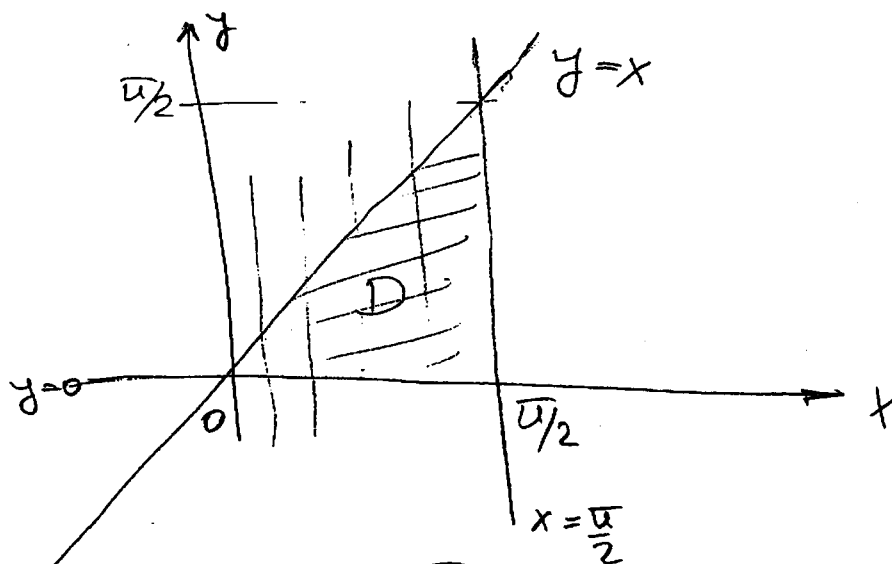


$$= \int_1^2 dy \int_0^y (x^2 + y^2) \, dx = \int_1^2 dy \left[\frac{x^3}{3} \Big|_0^y + y^2 x \Big|_0^y \right] =$$

$$= \int_1^2 \left(\frac{y^3}{3} + \frac{1}{3} y^3 \right) dy = \frac{4}{3} \int_1^2 y^3 \, dy = \frac{4}{3} \frac{y^4}{4} \Big|_1^2 =$$

$$= \frac{1}{3} (16 - 1) = \underline{\underline{\frac{15}{3}}}$$

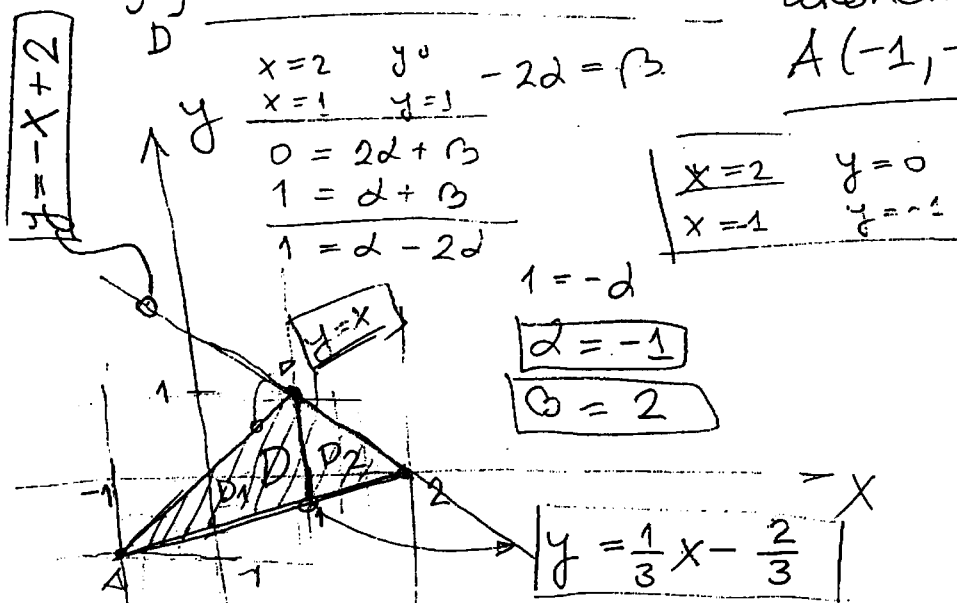
7. $\iint_D \sin(x+y) dx dy$ D отпавлена ститијаша
 $y=0, x=\frac{\pi}{2}, x=y$



$$\begin{aligned}
 \int_0^{\pi/2} dx \int_0^x \sin(x+y) dy &= \int_0^{\pi/2} dx \int_0^x (\sin x \cos y + \cos x \sin y) dy \\
 &= \int_0^{\pi/2} dx \left[\int_0^x \sin x \cos y dy + \int_0^x \cos x \sin y dy \right] = \\
 &= \int_0^{\pi/2} \left[\sin x \sin y \Big|_0^x + \cos x \cos y \Big|_0^x \right] dx = \\
 &= \int_0^{\pi/2} \left[\sin x (\sin x - \sin 0) - \cos x (\cos x - \cos 0) \right] dx = \\
 &= \int_0^{\pi/2} \left[\sin^2 x - \cos^2 x + \cos x \right] dx = \\
 &= \int_0^{\pi/2} \left[\sin^2 x - (1 - \sin^2 x) + \cos x \right] dx = \\
 &= \int_0^{\pi/2} \left[2\sin^2 x + \cos x - 1 \right] dx = 4
 \end{aligned}$$

$$\begin{aligned}
 &= 2 \int_0^{\sqrt{u}} \sin^2 x \, dx + \int_0^{\sqrt{u}} \cos x \, dx + \int_0^{\sqrt{u}} dx = \\
 &= 2 \left(\frac{\cos x \sin x}{2} \Big|_0^{\sqrt{u}} + \frac{1}{2} \int_0^{\sqrt{u}} \sin x \, dx \right) + \sin x \Big|_0^{\sqrt{u}} - x \Big|_0^{\sqrt{u}} = \\
 &= \left(\cancel{\cos \frac{\sqrt{u}}{2} \sin \frac{\sqrt{u}}{2}} - \cancel{\cos 0 \sin 0} \right) - \frac{1}{2} (\cancel{\cos \frac{\sqrt{u}}{2}} - \cancel{\cos 0}) + (\cancel{\sin \frac{\sqrt{u}}{2}} - \cancel{\sin 0}) \\
 &\quad - \left(\frac{\sqrt{u}}{2} - 0 \right) = \frac{1}{2} + \frac{1}{2} \frac{\sqrt{u}}{2} = \frac{3+2\sqrt{u}}{2}
 \end{aligned}$$

10. $\iint_D (2x+3y+1) \, dx \, dy$ D је одређена са
укупношћу:
 $A(-1,-1), B(2,0), C(1,1)$



$$\begin{aligned}
 I &= I_1 + I_2 \\
 \iint_D &= \iint_{D_1} + \iint_{D_2}
 \end{aligned}$$

$$\begin{aligned}
 y &= \alpha x + \beta \\
 0 &= 2\alpha + \beta \\
 -1 &= -\alpha + \beta \\
 -2\alpha &= \beta \\
 -1 &= -\alpha - 2\alpha / (-1)
 \end{aligned}$$

$$\begin{aligned}
 1 &= 3\alpha \\
 \alpha &= \frac{1}{3} \\
 \beta &= -\frac{2}{3}
 \end{aligned}$$

$$3y = x - 2$$

$$3y = x - 2$$

$$x = 3y + 2$$

$$\begin{aligned}
 x &= -y + 2 \\
 x &= y
 \end{aligned}$$

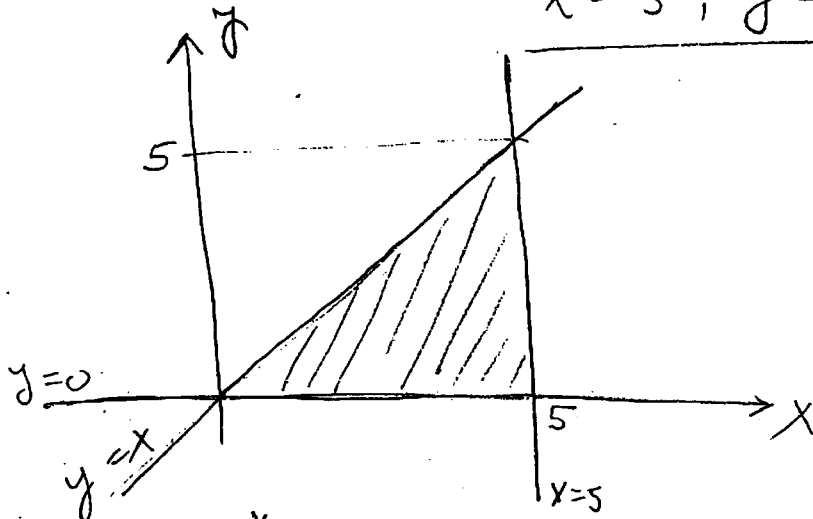
5

$$\begin{aligned}
I_1 &= \int_{-1}^1 dx \int_{\frac{x-2}{3}}^x (2x+3y+1) dy = \frac{9x^2 - x^2 + 2x + 4}{4 \cdot 9} \\
&= \int_{-1}^1 dx \left[2xy \Big|_{\frac{x-2}{3}}^x + 3 \frac{y^2}{2} \Big|_{\frac{x-2}{3}}^x + y \Big|_{\frac{x-2}{3}}^x \right] = \frac{3x - x + 2}{3} \frac{2x+2}{3} \\
&= \int_{-1}^1 dx \left[2x \left[x - \frac{x-2}{3} \right] + \frac{3}{2} \left[x^2 - \frac{(x-2)^2}{9} \right] + \left[x - \frac{x-2}{3} \right] \right] = \\
&= \int_{-1}^1 \left[\frac{4}{3} (x+1)x + \frac{1}{3} (4x^2 + x - 2) - \frac{2}{3} (x+1) \right] dx = \\
&= \frac{4}{3} \int_{-1}^1 (x+1)x dx + \frac{1}{3} \int_{-1}^1 (4x^2 + x - 2) dx - \frac{2}{3} \int_{-1}^1 (x+1) dx = \\
&= \frac{4}{3} \left(\frac{x^3}{3} \Big|_{-1}^1 + \frac{x^2}{2} \Big|_{-1}^1 \right) + \frac{1}{3} \left(4 \frac{x^3}{3} \Big|_{-1}^1 + \frac{x^2}{2} \Big|_{-1}^1 - 2x \Big|_{-1}^1 \right) - \\
&\quad - \frac{2}{3} \left(\frac{x^2}{2} \Big|_{-1}^1 + x \Big|_{-1}^1 \right) = \\
&= \frac{4}{3} \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{2} - \frac{1}{2} \right) + \frac{1}{3} \left(\frac{4}{3} + \frac{4}{3} + \frac{1}{2} - \frac{1}{2} - 2(1+1) \right) - \\
&\quad - \frac{2}{3} \left(\frac{1}{2} - \frac{1}{2} + 1 + 1 \right) = \frac{8}{9} + \frac{8}{9} - \left(\frac{4}{3} - \frac{4}{3} \right) = \frac{16 - 24}{9} = -\frac{8}{9}
\end{aligned}$$

6.7

$$\begin{aligned}
 I_2 &= \int_1^2 dx \int_{\frac{x-2}{3}}^{2-x} (2x+3y+1) dy = \\
 &= \int_1^2 dx \left[2x \cdot y \Big|_{\frac{x-2}{3}}^{2-x} + \frac{3}{2} y^2 \Big|_{\frac{x-2}{3}}^{2-x} + \left(y \Big|_{\frac{x-2}{3}}^{2-x} \right) \right] \\
 &= \int_1^2 \left[2x \left((2-x) - \frac{x-2}{3} \right) + \frac{3}{2} \left((2-x)^2 - \frac{(x-2)^2}{9} \right) + \left(2-x - \frac{x-2}{3} \right) \right] dx = \\
 &= \int_1^2 \left[\frac{8}{3} (2x-x^2) + \frac{4}{3} (x^2-2x+4) + \frac{4}{3} (2-x) \right] dx = \\
 &= \frac{8}{3} \left(2 \frac{x^2}{2} \Big|_1^2 - \frac{x^3}{3} \Big|_1^2 \right) + \frac{4}{3} \left(\frac{x^3}{3} \Big|_1^2 - 2 \frac{x^2}{2} \Big|_1^2 + 4x \Big|_1^2 \right) + \\
 &\quad + \frac{4}{3} \left(2x \Big|_1^2 - \frac{x^2}{2} \Big|_1^2 \right) = \frac{1}{3} - 3 + 4 = \frac{1-9+12}{3} = \frac{4}{3} \\
 &= \frac{8}{3} \left(4 - 1 - \left(\frac{8}{3} + \frac{1}{3} \right) \right) + \frac{4}{3} \left(\frac{2}{3} - \frac{1}{3} - (4-1) + 4(2-1) \right) + \\
 &\quad + \frac{4}{3} \left(2(2-1) - \left(\frac{4}{2} - \frac{1}{2} \right) \right) = 2 - \frac{3}{2} = \frac{4-3}{2} = \frac{1}{2} \\
 &= \frac{16}{9} + \frac{16}{9} + \frac{2}{3} = \frac{16+16+6}{9} = \frac{38}{9} \\
 \boxed{I} &= -\frac{8}{9} + \frac{38}{9} = \boxed{\frac{30}{9}} \quad \checkmark
 \end{aligned}$$

12. $\iint_D (x+2y) dx dy$, D ограничена линиями
 $x=5, y=0, y=x$

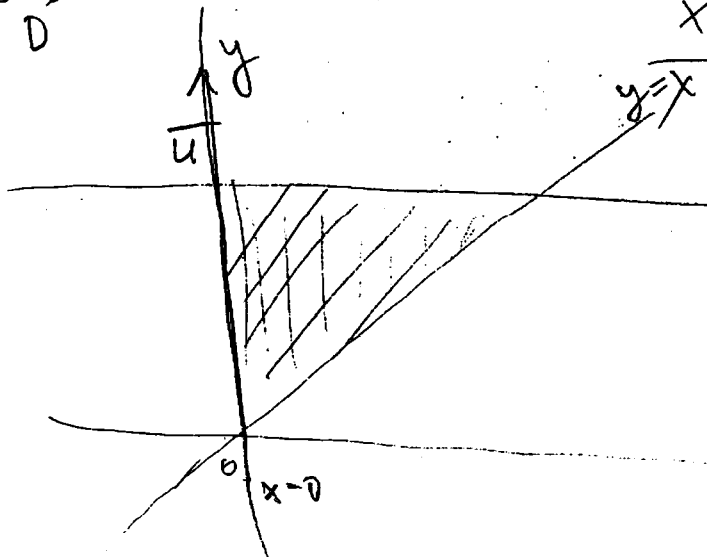


$$I = \int_0^5 dx \int_0^x (x+2y) dy = \int_0^5 dx \left[xy \Big|_0^x + \frac{y^2}{2} \Big|_0^x \right]$$

$$= \int_0^5 dx (x^2 + x^2) = \int_0^5 2x^2 dx$$

$$I = \frac{2}{3} x^3 \Big|_0^5 = \frac{2}{3} (125 - 0) = \frac{250}{3}$$

14. $\iint_D \cos(x+y) dx dy$, D ограничена линиями
 $x=0, x=y, y=5$



> x

8

$$I = \int_0^{\pi} dy \int_0^y \cos(x+y) dx =$$

$$= \int_0^{\pi} dy \int_0^y (\cos x \cos y - \sin x \sin y) dx =$$

$$= \int_0^{\pi} dy \left[\cos y \cdot \sin x \Big|_0^y + \sin y \cdot \cos x \Big|_0^y \right] =$$

$$= \int_0^{\pi} \left(\cos y \sin y - \cos y \cdot \sin 0 \right) + \left(\sin y \cos y - \sin y \cos 0 \right) dy$$

$\frac{\sin 2y}{2 \sin x \cos x}$

$$I = \int_0^{\pi} (\cos y \sin y + \sin y \cos y - \sin y) dy$$

$$I = \int_0^{\pi} \sin 2y dy - \int_0^{\pi} \sin y dy =$$

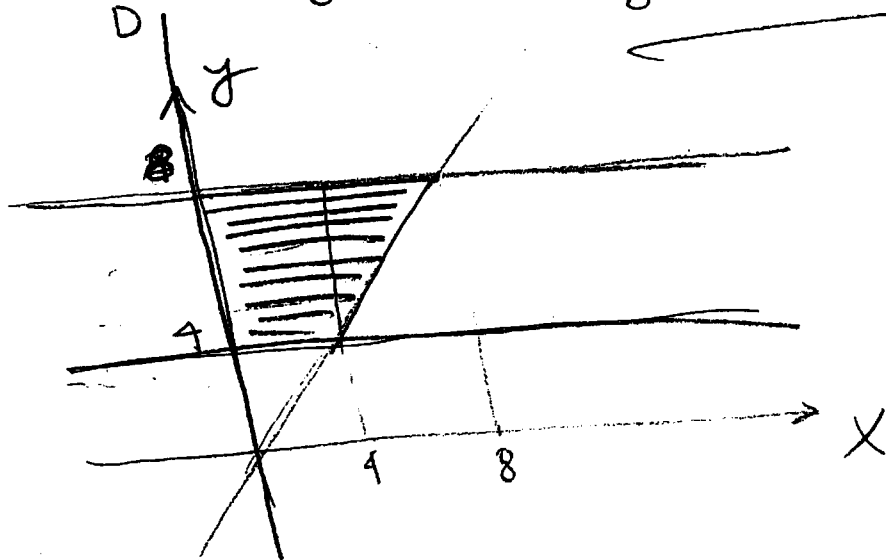
$$= -\frac{1}{2} \cos 2y \Big|_0^{\pi} + \cos y \Big|_0^{\pi} = -1 + 1$$

$$I = -\frac{1}{2} (\cos 2\pi - \cos 0) + (\cos \pi - \cos 0)$$

$$\boxed{I = -2} ?$$

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✓
 (16) $\iint_D \frac{y^3}{x^2+y^2} dx dy$, Додатково ступінь
 $y=4, y=8, y=x, x=0$



$$\frac{1}{x^2+a^2}$$

$$I = \int_4^8 dy \int_0^y \frac{y^3}{x^2+y^2} dx = \int_4^8 y^3 dy \int_0^y \frac{1}{x^2+y^2} dx =$$

$$= \int_4^8 y^3 dy \cdot \left(\frac{1}{y} \arctg x \Big|_0^y \right) =$$

$$I = \int_4^8 y^3 dy \cdot \left(\frac{1}{y} (\arctg y - \arctg 0) \right) =$$

$$= \int_4^8 y^2 \arctg y dy = \begin{cases} u = \arctg y \rightarrow du = \frac{1}{1+y^2} dy \\ v = \frac{y^3}{3} \end{cases}$$

$$= \frac{y^3}{3} \arctg y \Big|_4^8 - \int_4^8 \frac{1}{1+y^2} \cdot \frac{y^3}{3} dy$$

100

1998, 1999, 2000, 2001, 2002, 2003, 2004, 2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020, 2021, 2022, 2023, 2024, 2025, 2026, 2027, 2028, 2029, 2030, 2031, 2032, 2033, 2034, 2035, 2036, 2037, 2038, 2039, 2040, 2041, 2042, 2043, 2044, 2045, 2046, 2047, 2048, 2049, 2050, 2051, 2052, 2053, 2054, 2055, 2056, 2057, 2058, 2059, 2060, 2061, 2062, 2063, 2064, 2065, 2066, 2067, 2068, 2069, 2070, 2071, 2072, 2073, 2074, 2075, 2076, 2077, 2078, 2079, 2080, 2081, 2082, 2083, 2084, 2085, 2086, 2087, 2088, 2089, 2090, 2091, 2092, 2093, 2094, 2095, 2096, 2097, 2098, 2099, 2100, 2101, 2102, 2103, 2104, 2105, 2106, 2107, 2108, 2109, 2110, 2111, 2112, 2113, 2114, 2115, 2116, 2117, 2118, 2119, 2120, 2121, 2122, 2123, 2124, 2125, 2126, 2127, 2128, 2129, 2130, 2131, 2132, 2133, 2134, 2135, 2136, 2137, 2138, 2139, 2140, 2141, 2142, 2143, 2144, 2145, 2146, 2147, 2148, 2149, 2150, 2151, 2152, 2153, 2154, 2155, 2156, 2157, 2158, 2159, 2160, 2161, 2162, 2163, 2164, 2165, 2166, 2167, 2168, 2169, 2170, 2171, 2172, 2173, 2174, 2175, 2176, 2177, 2178, 2179, 2180, 2181, 2182, 2183, 2184, 2185, 2186, 2187, 2188, 2189, 2190, 2191, 2192, 2193, 2194, 2195, 2196, 2197, 2198, 2199, 2200, 2201, 2202, 2203, 2204, 2205, 2206, 2207, 2208, 2209, 2210, 2211, 2212, 2213, 2214, 2215, 2216, 2217, 2218, 2219, 2220, 2221, 2222, 2223, 2224, 2225, 2226, 2227, 2228, 2229, 2230, 2231, 2232, 2233, 2234, 2235, 2236, 2237, 2238, 2239, 2240, 2241, 2242, 2243, 2244, 2245, 2246, 2247, 2248, 2249, 2250, 2251, 2252, 2253, 2254, 2255, 2256, 2257, 2258, 2259, 2260, 2261, 2262, 2263, 2264, 2265, 2266, 2267, 2268, 2269, 2270, 2271, 2272, 2273, 2274, 2275, 2276, 2277, 2278, 2279, 2280, 2281, 2282, 2283, 2284, 2285, 2286, 2287, 2288, 2289, 2290, 2291, 2292, 2293, 2294, 2295, 2296, 2297, 2298, 2299, 2300, 2301, 2302, 2303, 2304, 2305, 2306, 2307, 2308, 2309, 2310, 2311, 2312, 2313, 2314, 2315, 2316, 2317, 2318, 2319, 2320, 2321, 2322, 2323, 2324, 2325, 2326, 2327, 2328, 2329, 2330, 2331, 2332, 2333, 2334, 2335, 2336, 2337, 2338, 2339, 2340, 2341, 2342, 2343, 2344, 2345, 2346, 2347, 2348, 2349, 2350, 2351, 2352, 2353, 2354, 2355, 2356, 2357, 2358, 2359, 2360, 2361, 2362, 2363, 2364, 2365, 2366, 2367, 2368, 2369, 2370, 2371, 2372, 2373, 2374, 2375, 2376, 2377, 2378, 2379, 2380, 2381, 2382, 2383, 2384, 2385, 2386, 2387, 2388, 2389, 2390, 2391, 2392, 2393, 2394, 2395, 2396, 2397, 2398, 2399, 2400, 2401, 2402, 2403, 2404, 2405, 2406, 2407, 2408, 2409, 2410, 2411, 2412, 2413, 2414, 2415, 2416, 2417, 2418, 2419, 2420, 2421, 2422, 2423, 2424, 2425, 2426, 2427, 2428, 2429, 2430, 2431, 2432, 2433, 2434, 2435, 2436, 2437, 2438, 2439, 2440, 2441, 2442, 2443, 2444, 2445, 2446, 2447, 2448, 2449, 2450, 2451, 2452, 2453, 2454, 2455, 2456, 2457, 2458, 2459, 2460, 2461, 2462, 2463, 2464, 2465, 2466, 2467, 2468, 2469, 2470, 2471, 2472, 2473, 2474, 2475, 2476, 2477, 2478, 2479, 2480, 2481, 2482, 2483, 2484, 2485, 2486, 2487, 2488, 2489, 2490, 2491, 2492, 2493, 2494, 2495, 2496, 2497, 2498, 2499, 2500, 2501, 2502, 2503, 2504, 2505, 2506, 2507, 2508, 2509, 2510, 2511, 2512, 2513, 2514, 2515, 2516, 2517, 2518, 2519, 2520, 2521, 2522, 2523, 2524, 2525, 2526, 2527, 2528, 2529, 2530, 2531, 2532, 2533, 2534, 2535, 2536, 2537, 2538, 2539, 2540, 2541, 2542, 2543, 2544, 2545, 2546, 2547, 2548, 2549, 2550, 2551, 2552, 2553, 2554, 2555, 2556, 2557, 2558, 2559, 2560, 2561, 2562, 2563, 2564, 2565, 2566, 2567, 2568, 2569, 2570, 2571, 2572, 2573, 2574, 2575, 2576, 2577, 2578, 2579, 2580, 2581, 2582, 2583, 2584, 2585, 2586, 2587, 2588, 2589, 2590, 2591, 2592, 2593, 2594, 2595, 2596, 2597, 2598, 2599, 2600, 2601, 2602, 2603, 2604, 2605, 2606, 2607, 2608, 2609, 2610, 2611, 2612, 2613, 2614, 2615, 2616, 2617, 2618, 2619, 2620, 2621, 2622, 2623, 2624, 2625, 2626, 2627, 2628, 2629, 2630, 2631, 2632, 2633, 2634, 2635, 2636, 2637, 2638, 2639, 2640, 2641, 2642, 2643, 2644, 2645, 2646, 2647, 2648, 2649, 2650, 2651, 2652, 2653, 2654, 2655, 2656, 2657, 2658, 2659, 2660, 2661, 2662, 2663, 2664, 2665, 2666, 2667, 2668, 2669, 2670, 2671, 2672, 2673, 2674, 2675, 2676, 2677, 2678, 2679,

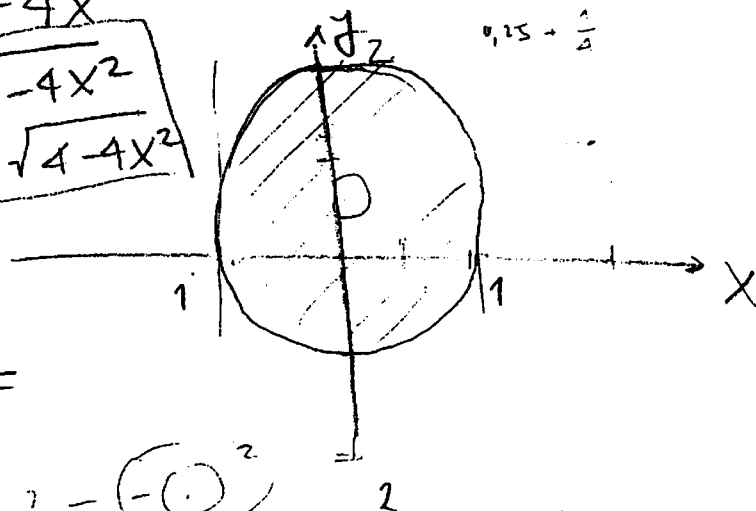
the 1990s, the number of people in the world who are undernourished has declined from 1.1 billion to 800 million. The number of people who are malnourished has declined from 1.5 billion to 1 billion. The number of people who are obese has increased from 100 million to 300 million. The number of people who are overweight has increased from 100 million to 300 million. The number of people who are obese and overweight has increased from 100 million to 300 million. The number of people who are obese and overweight has increased from 100 million to 300 million.

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$$\begin{aligned}
 \frac{1}{3} \int_4^8 \frac{y^3}{1+y^2} dy &= \frac{1}{3} \int_4^8 \frac{(1+y^2)y - y}{1+y^2} dy = \\
 &= \frac{1}{3} \left. \frac{y^2}{2} \right|_4^8 - \frac{1}{3} \int_4^8 \frac{y}{1+y^2} dy = \begin{cases} 1+y^2=t \\ 2y dy = dt \end{cases} \\
 &= \frac{1}{3} \left. \frac{y^2}{2} \right|_4^8 - \frac{1}{3} \cdot \frac{1}{2} \ln |1+y^2| \Big|_4^8 = \\
 &= \frac{1}{6} (64 - 16) - \frac{1}{6} (\ln 65 - \ln 17) \\
 &= \boxed{8 - \frac{1}{6} \ln \frac{65}{17}} \quad \checkmark
 \end{aligned}$$

(18) $\iint_D xy \, dx \, dy$, $D = \{(x, y) \mid 4x^2 + y^2 \leq 4\}$

$$\begin{aligned}
 y^2 &= 4 - 4x^2 \\
 y_1 &= \sqrt{4 - 4x^2} \\
 y_2 &= -\sqrt{4 - 4x^2}
 \end{aligned}$$

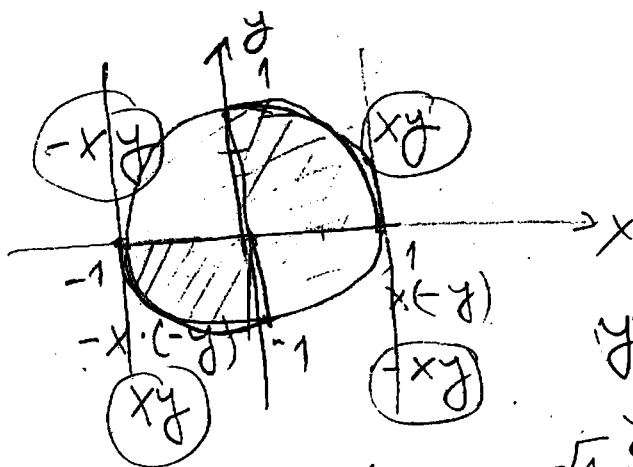


$$\int_{-1}^1 dx \int_{-\sqrt{4-4x^2}}^{\sqrt{4-4x^2}} xy \, dy =$$

$$\begin{aligned}
 &= \int_{-1}^1 x dx \cdot \left. \frac{y^2}{2} \right|_{-\sqrt{4-4x^2}}^{\sqrt{4-4x^2}} = \frac{1}{2} \int_{-1}^1 x (4 - 4x^2 - 4 + 4x^2) dx \\
 &\quad \text{?}
 \end{aligned}$$

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20. $\iint_D |xy| dx dy, D = \{(x,y) | x^2 + y^2 \leq 1\}$



$$I = \int_{-1}^1 dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} |xy| dy = 2 \int_{-1}^1 dx \cdot 2 \int_0^{\sqrt{1-x^2}} xy dy \quad ?$$

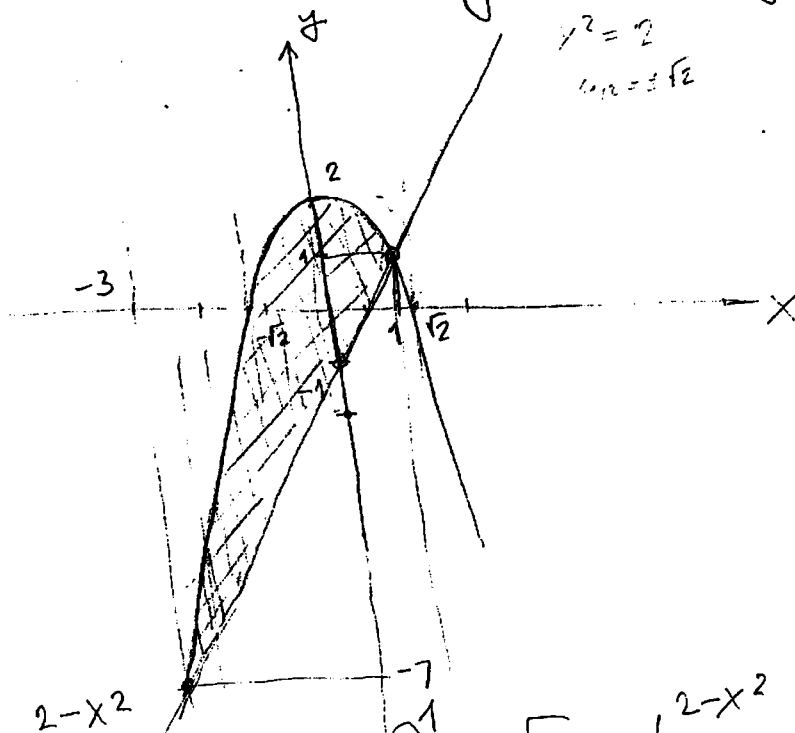
$y^2 = 1 - x^2$
 $y = \pm \sqrt{1-x^2}$

21. $\iint_D (x-y) dx dy, D$ ограничена кривыми

$$y = 2 - x^2, y = 2x - 1$$

$x^2 = 2$
 $x = \pm \sqrt{2}$

$-6 \quad 2x = 1$
 $x = \frac{1}{2}$



$$\begin{aligned} 2 - x^2 &= 2x - 1 \\ 2 + 1 &= 2x + x^2 \\ x^2 + 2x - 3 &= 0 \\ x_{1,2} &= \frac{-2 \pm \sqrt{4 + 12}}{2} \\ x_{1,2} &= \frac{-2 \pm 4}{2} \end{aligned}$$

$$I = \int_{-3}^1 dx \int_{2x-1}^{2-x^2} (x-y) dy = \int_{-3}^1 dx \left[xy \Big|_{2x-1}^{2-x^2} - \frac{y^2}{2} \Big|_{2x-1}^{2-x^2} \right] = 15$$

$$I = \int_{-\sqrt{R}}^{\sqrt{R}} dx \int_{-\sqrt{R-x^2}}^{\sqrt{R-x^2}} y^2 \sqrt{R^2-x^2} dy =$$

$$= \int_{-\sqrt{R}}^{\sqrt{R}} \sqrt{R^2-x^2} dx \int_{-\sqrt{R-x^2}}^{\sqrt{R-x^2}} y^2 dy \Rightarrow$$

$$I = \int_{-\sqrt{R}}^{\sqrt{R}} \sqrt{R^2-x^2} dx \left(\frac{y^3}{3} \Big|_{-\sqrt{R-x^2}}^{\sqrt{R-x^2}} \right) = \quad ?$$

$$= \frac{1}{3} \int_{-\sqrt{R}}^{\sqrt{R}} \left(\sqrt{R-x^2}^3 + \sqrt{R-x^2}^3 \right) \sqrt{R^2-x^2} dx = \frac{(R-x^2)(R^2-x^2)}{R^3 - R^2x^2 - x^2R + x^4}$$

$$= \frac{1}{3} \int_{-\sqrt{R}}^{\sqrt{R}} 2 \sqrt{R-x^2}^3 \sqrt{R^2-x^2} dx = \frac{2}{3} \int_{-\sqrt{R}}^{\sqrt{R}} (R-x^2) \sqrt{R-x^2} \sqrt{R^2-x^2} dx$$

$$I = \int_1^2 dx \int_{\frac{1}{x}}^{\sqrt{x}} y \ln x dy = \int_1^2 \ln x dx \int_{\frac{1}{x}}^{\sqrt{x}} y dy =$$

$$I = \int_1^2 \ln x dx \left[\frac{y^2}{2} \right]_{\frac{1}{x}}^{\sqrt{x}} = \frac{1}{2} \int_1^2 \left(x - \frac{1}{x} \right) \ln x dx =$$

$$= \frac{1}{2} \int_1^2 x \ln x dx - \frac{1}{2} \int_1^2 \frac{\ln x dx}{x^2} = \frac{x^{-2+1}}{-2+1}$$

$$= \frac{1}{2} \cdot \left\{ \begin{array}{l} u = \ln x \rightarrow du = \frac{1}{x} dx \\ x^2 = v \end{array} \right\} - \frac{1}{2} \cdot \left\{ \begin{array}{l} u = \ln x \rightarrow du = \frac{1}{x} dx \\ v = -\frac{1}{x} \end{array} \right\} =$$

$$= \frac{1}{2} \left[\frac{x^2}{2} \ln x \right]_1^2 - \frac{1}{2} \int_1^2 x dx - \frac{1}{2} \left[-\frac{1}{x} \ln x \right]_1^2 + \int_1^2 \frac{1}{x^2} dx =$$

$$= \frac{x^2}{4} \ln x \Big|_1^2 - \frac{1}{4} \cdot \frac{x^2}{2} \Big|_1^2 + \frac{1}{2x} \ln x \Big|_1^2 + \frac{1}{2} \frac{1}{x} \Big|_1^2 =$$

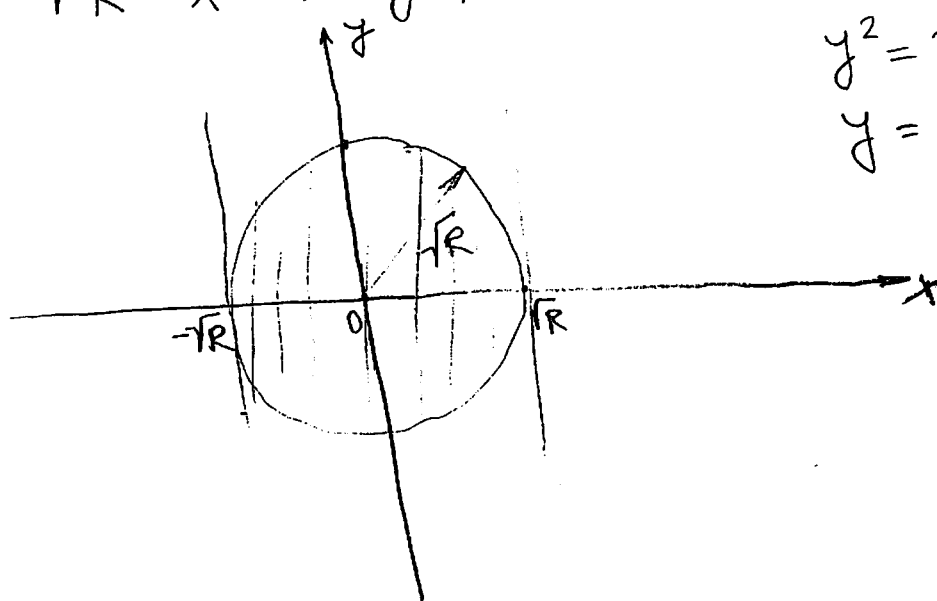
$$= \frac{1}{4} (4 \ln 2 - \ln 1) - \frac{1}{8} (4 - 1) + \frac{1}{2} \left(\frac{\ln 2}{2} - \ln 1 \right) + \frac{1}{2} \left(\frac{1}{2} - 1 \right) =$$

$$= \ln 2 - \frac{3}{8} + \frac{\ln 2}{4} - \frac{1}{4} = \frac{5}{4} \ln 2 - \frac{5}{8}$$

$$(26) \iint_D y^4 \sqrt{R^2 - x^2} dx dy, D = \{(x, y) | x^2 + y^2 \leq R^2\}$$

$$y^2 = R^2 - x^2$$

$$y = \pm \sqrt{R^2 - x^2}$$



$$I = \int_{-3}^1 \left[x(2-x^2-2x+1) - \frac{1}{2} \left(\frac{(2-x^2)^2 - (2x-1)^2}{4-2x^2+x^4 - (4x^2-4x+1)} \right) \right] dx = \quad 6.14$$

$$I = \int_{-3}^1 \left(-x^3 - 2x^2 + 3x - \frac{1}{2} \left(\frac{4-2x^2+x^4-4x^2+4x-1}{x^4-6x^2+4x+3} \right) \right) dx$$

$$I = \int_{-3}^1 \left(-x^3 - 2x^2 + 3x - \frac{x^4}{2} + 3x^2 - 2x + \frac{3}{2} \right) dx$$

$$I = \int_{-3}^1 \left(-\frac{x^4}{2} - x^3 + x^2 + x + \frac{3}{2} \right) dx =$$

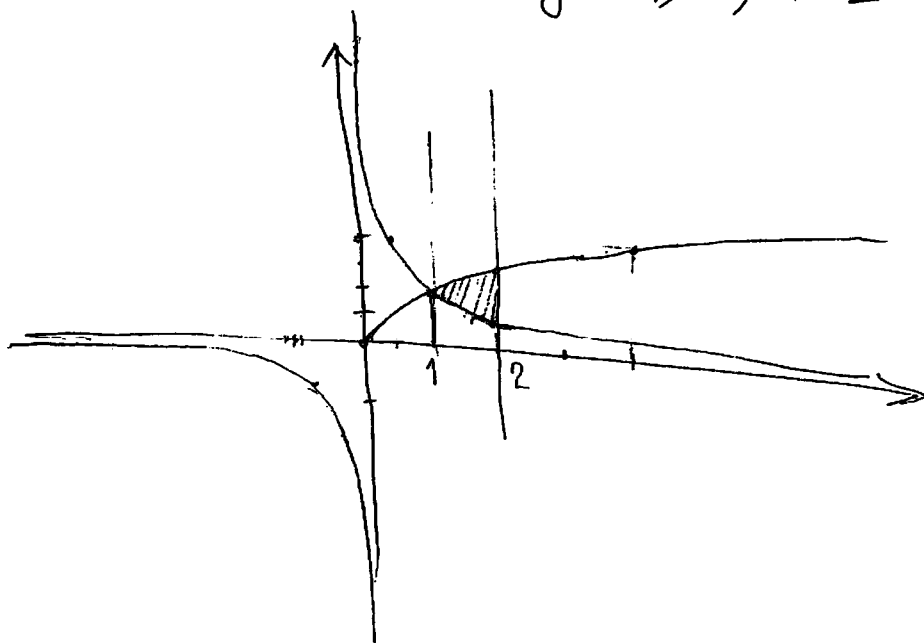
$$= -\frac{1}{2} \frac{x^5}{5} \Big|_{-3}^1 - \frac{x^4}{4} \Big|_{-3}^1 + \frac{x^3}{3} \Big|_{-3}^1 + \frac{x^2}{2} \Big|_{-3}^1 + \frac{3}{2} x \Big|_{-3}^1 =$$

$$= -\frac{1}{10} (1+243) - \frac{1}{4} (1-81) + \frac{1}{3} (1+27) + \frac{1}{2} (1-9) + \frac{3}{2} (1+3) = -\frac{244}{10} - \frac{80}{4} + \frac{28}{3} + \frac{8}{2} + \frac{12}{2}$$

$$\boxed{I} = \frac{2928 - 2400 + 1120 - 480 + 720}{120} = \frac{1888}{120} = \frac{472}{30} = \underline{\underline{\frac{236}{15}}}$$

(24.) $\iint_D y \ln x \, dx \, dy$, D отграницата елиптична $xy=1$
 $y = \sqrt{x}$, $x=2$

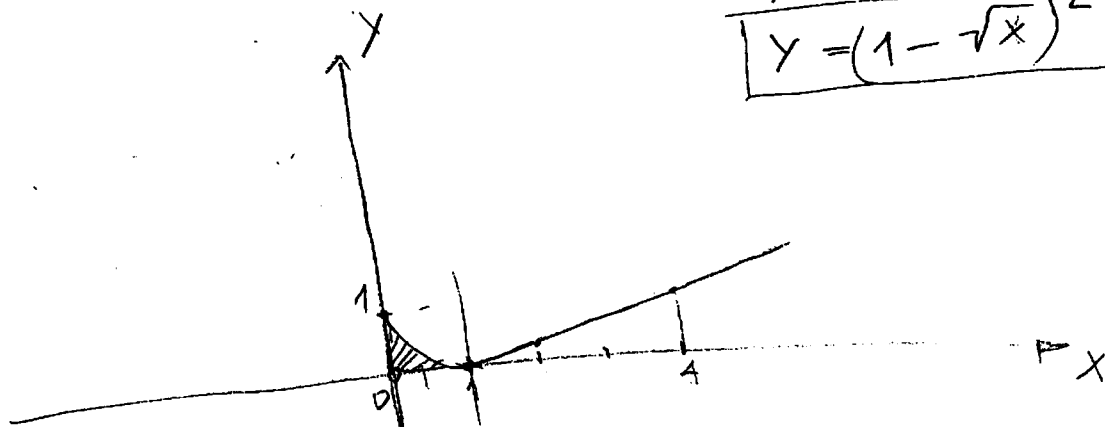
$$y = \frac{1}{x}$$



28. $\iint_D xy \, dx \, dy$, Двопративна координатна осакна и одредбува $\sqrt{x} + \sqrt{y} = 1$

$$\sqrt{y} = 1 - \sqrt{x}$$

$$y = (1 - \sqrt{x})^2$$



$$I = \int_0^1 x \, dx \int_0^{(1-\sqrt{x})^2} y \, dy = \int_0^1 x \, dx \cdot \frac{y^2}{2} \Big|_0^{(1-\sqrt{x})^2} = \frac{1}{2} \int_0^1 x (1-\sqrt{x})^4 \, dx$$

$$(1-\sqrt{x})(1-\sqrt{x}) = (1-2\sqrt{x}+x)(1-2\sqrt{x}+x) = 1 - 2\sqrt{x} + x - 2\sqrt{x} + 4x - 2\sqrt{x^3} + x - 2\sqrt{x^3} + x^2$$

$$\frac{x^{\frac{3}{2}+1} = \frac{5}{2}}{\frac{3}{2}+1 = \frac{5}{2}}$$

$$\frac{x^{\frac{5}{2}+1} = \frac{7}{2}}{\frac{5}{2}+1 = \frac{7}{2}}$$

$$\frac{1-4\sqrt{x}+6x-4\sqrt{x^3}+x^2}{1-4\sqrt{x}+6x-4\sqrt{x^3}+x^2}$$

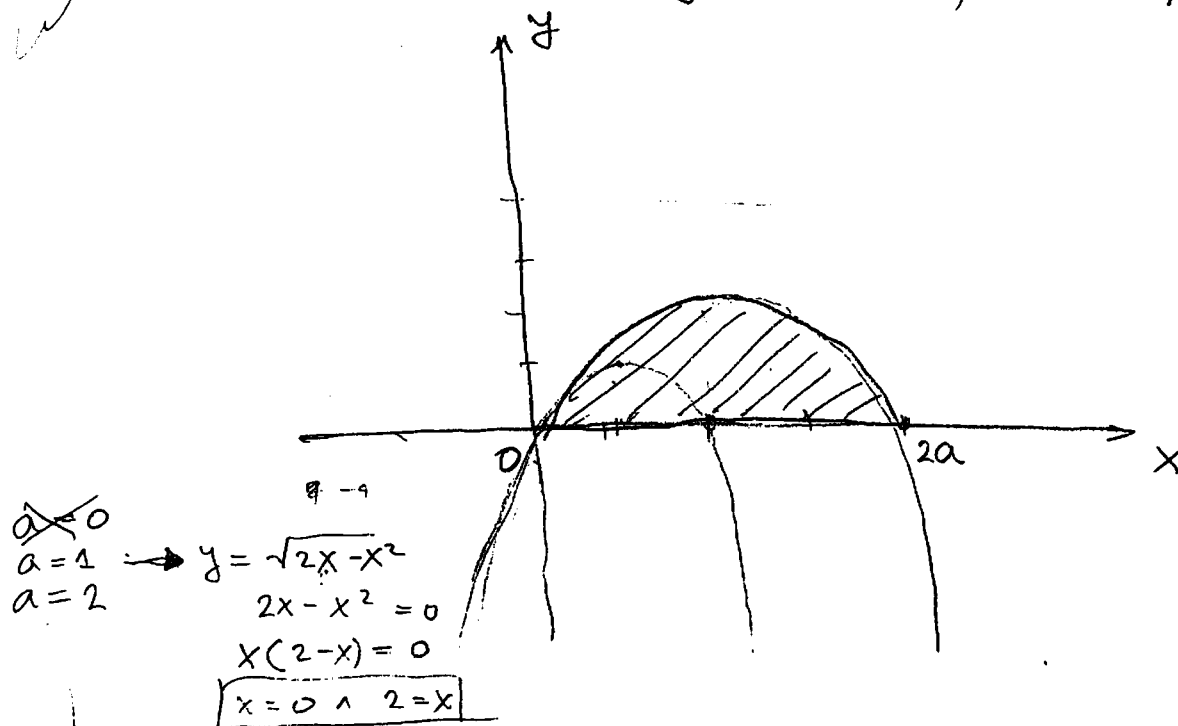
$$I = \frac{1}{2} \int_0^1 (x - 4\sqrt{x^3} + 6x^2 - 4\sqrt{x^5} + x^3) \, dx =$$

$$= \frac{1}{2} \left(\frac{x^2}{2} \Big|_0^1 - 4 \cdot \frac{2}{5} x^{\frac{5}{2}} \Big|_0^1 + 6 \cdot \frac{x^3}{3} \Big|_0^1 - 4 \cdot \frac{2}{7} x^{\frac{7}{2}} \Big|_0^1 + \frac{x^4}{4} \Big|_0^1 \right) =$$

$$= \frac{1}{4} (1-0) - \frac{4}{5} (1-0) + (1-0) - \frac{4}{7} (1-0) + \frac{1}{8} (1-0) =$$

$$= \frac{1}{4} - \frac{4}{5} + 1 - \frac{4}{7} + \frac{1}{8} = \frac{70-244+280-160+35}{280} = \frac{-19}{280}$$

32. $\int_0^{\sqrt{2ax-x^2}} \int_0^y x^2 y \, dx \, dy$, D область ограничена $y=0$
 $y = \sqrt{2ax-x^2}$ $a \neq 0$



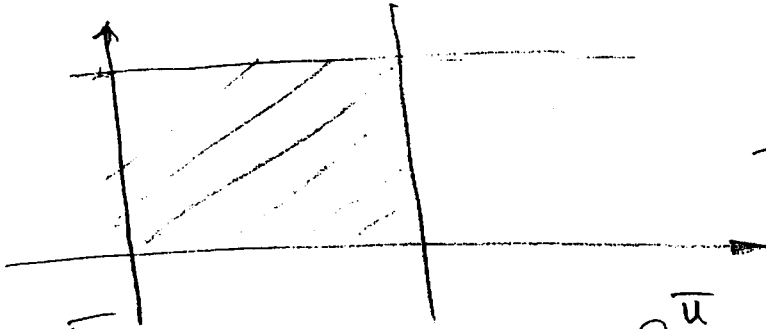
$y = \sqrt{4x-x^2}$ $\rightarrow x_1=0 \wedge x_2=4$
 $x_1=0 \wedge x_2=2a$

$$I = \int_0^{2a} x^2 dx \int_0^{\sqrt{2ax-x^2}} y \, dy = \int_0^{2a} x^2 dx \cdot \frac{1}{2} (2ax-x^2) =$$

$$= \frac{1}{2} \int_0^{2a} (2ax^3 - x^4) dx = a \int_0^{2a} x^3 dx - \frac{1}{2} \int_0^{2a} x^4 dx =$$

$$= \dots$$

37) $\iint_D |\cos(x+y)| dx dy$ $D = \{(x,y) | 0 \leq x \leq \bar{u}, 0 \leq y \leq \bar{u}\}$



$$I = \int_0^{\bar{u}} dx \int_0^{\bar{u}} |\cos(x+y)| dy = \int_0^{\bar{u}} dx \int_0^{\bar{u}} |\cos(x+y)| dy$$

$$I = \int_0^{\bar{u}} dx \int_0^{\bar{u}} (\cos x \cos y - \sin x \sin y) dy =$$

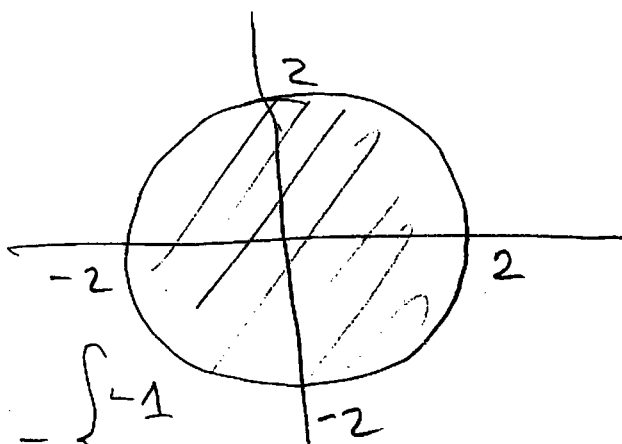
$$= \int_0^{\bar{u}} dx \left[\int_0^{\bar{u}} \cos x \cos y dy + \int_0^{\bar{u}} \sin x \sin y dy \right] =$$

$$= \int_0^{\bar{u}} dx \left[-\cos x \sin y \Big|_0^{\bar{u}} - \sin x \cos y \Big|_0^{\bar{u}} \right] =$$

$$= \int_0^{\bar{u}} dx (-\sin x (\cos \bar{u} - \cos 0) - \cos x (\sin \bar{u} - \sin 0)) = 2 \int_0^{\bar{u}} \sin x dx =$$

$$= -2 \cos x \Big|_0^{\bar{u}} = -2 (\cos \bar{u} - \cos 0) = \underline{\underline{4}}$$

$$(38) \iint_D \text{sgn}(x^2 - y^2 + 2) dx dy; D = \{(x, y) \mid x^2 + y^2 \leq 4\}$$



$$\text{sgn}(x^2 - y^2 + 2) = \begin{cases} 1 \\ -1 \end{cases}$$

$$x^2 - y^2 + 2 = 0$$

$$x^2 - y^2 = -2$$

$$x=0 \quad y=\pm\sqrt{2} \quad ?$$

$$\underline{y=0 \quad x=\pm\sqrt{2}}$$

6.2.a)

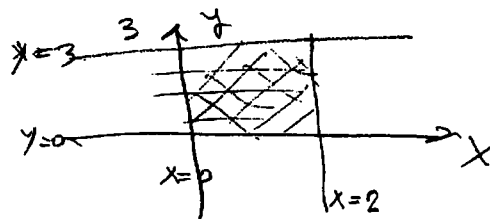
$$\textcircled{1.} \int_0^2 dx \int_0^3 (x^2 + 2xy) dy =$$

$$0 \leq x \leq 2$$

$$0 \leq y \leq 3$$

$$0 \leq y \leq 3$$

$$D: 0 \leq x \leq 2$$



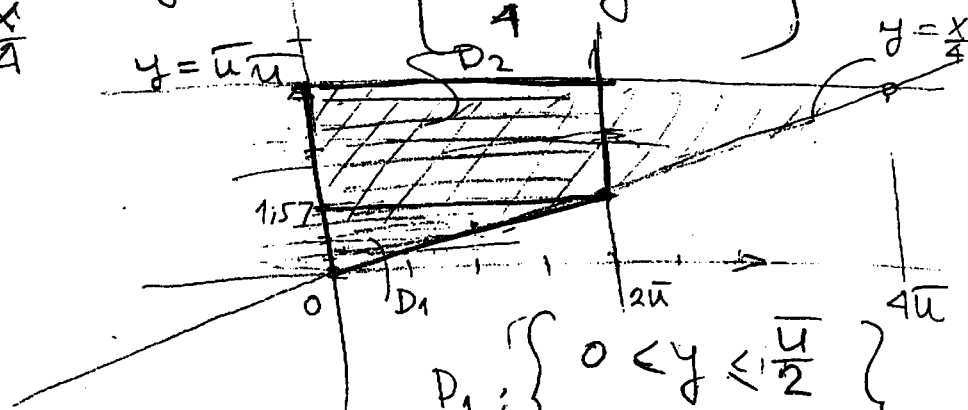
$$\int_0^3 dy \int_0^2 (x^2 + 2xy) dx = \dots \dots \dots \checkmark$$

$$\int_0^{2\bar{u}} \cos^2 x dx \int_{\frac{\bar{x}}{4}}^{\bar{u}} y dy \rightarrow D:$$

$$D: \begin{cases} 0 \leq x \leq 2\bar{u} \\ \frac{\bar{x}}{4} \leq y \leq \bar{u} \end{cases}$$

$$\frac{x}{4} = \bar{u}$$

$$x = 4\bar{u}$$



$$D_1: \begin{cases} 0 \leq y \leq \frac{\bar{u}}{2} \\ 0 \leq x \leq 4y \end{cases}$$

$$D_2: \begin{cases} \frac{\bar{u}}{2} \leq y \leq \bar{u} \\ 0 \leq x \leq 2\bar{u} \end{cases}$$

$$I = \int_0^{\bar{u}/2} y dy \int_0^{4y} \cos^2 x dx + \int_{\bar{u}/2}^{\bar{u}} y dy \int_0^{2\bar{u}} \cos^2 x dx =$$

$$I = \int_0^{\pi/2} y dy \left[\frac{\cos x \sin x}{2} \Big|_0^{4y} + \frac{1}{2} \int_0^{4y} \cos x dx \right] + \frac{\sin 2x}{4}$$

$$+ \int_{\pi/2}^{\pi} y dy \left[\frac{\cos x \sin x}{2} \Big|_0^{2u} + \frac{1}{2} \int_0^{2u} \cos x dx \right] =$$

$$= \int_0^{\pi/2} y dy \left[\frac{1}{2} (\cos 4y \sin 4y) + \frac{1}{2} (\sin 4y) \right] =$$

$$= \int_0^{\pi/2} \left(\frac{1}{4} \sin 8y + \frac{1}{2} \sin 4y \right) y dy =$$

$$= \frac{1}{4} \int_0^{\pi/2} y \sin 8y dy + \frac{1}{2} \int_0^{\pi/2} y \sin 4y dy =$$

$$\left\{ \begin{array}{l} u = y \Rightarrow du = dy \\ v = \frac{1}{8} \cos 8y \end{array} \right\}$$

$$= \frac{1}{4} \left(-\frac{1}{8} y \cos 8y \Big|_0^{\pi/2} + \frac{1}{8} \int_0^{\pi/2} \cos 8y dy \right) + \frac{1}{2} \left(-\frac{1}{4} y \cos 4y \Big|_0^{\pi/2} + \frac{1}{4} \int_0^{\pi/2} \cos 4y dy \right) =$$

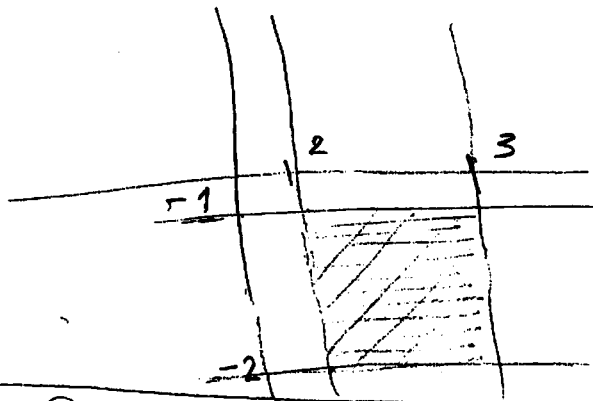
$$= -\frac{1}{32} \frac{\pi}{2} \cos 4\pi + \frac{1}{64} (\sin 4\pi - \sin 0) - \frac{1}{8} \frac{\pi}{2} \cos 2\pi +$$

$$+ \frac{1}{16} (\sin 2\pi - \sin 0) = -\frac{\pi}{64} - \frac{\pi}{16} = -\frac{5\pi}{64}$$

$$\textcircled{4} \int_{-2}^3 dx \int_{-2}^{-1} f(x,y) dy = \left\{ \begin{array}{l} 2 \leq x \leq 3 \\ -2 \leq y \leq -1 \end{array} \right\} =$$

$$\begin{array}{l} -2 \leq y \leq -1 \\ 2 \leq x \leq 3 \end{array}$$

$$I = \int_2^3 dy \int_{-2}^{-1} f(x,y) dx$$



$$\textcircled{5} \int_0^1 dx \int_0^{2-2x} f(x,y) dy = \left\{ \begin{array}{l} 0 \leq x \leq 1 \\ 0 \leq y \leq 2-2x \end{array} \right\} \Rightarrow$$

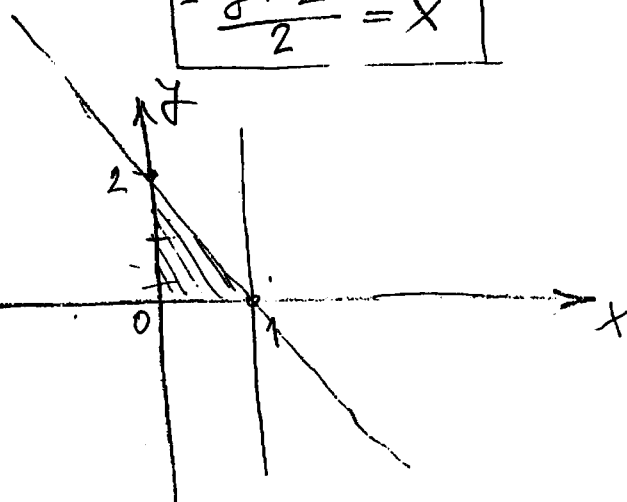
$$\Rightarrow \left\{ \begin{array}{l} 0 \leq y \leq 2 \\ 0 \leq x \leq -\frac{y+2}{2} \end{array} \right\}$$

$$y = 2 - 2x$$

$$y - 2 = -2x$$

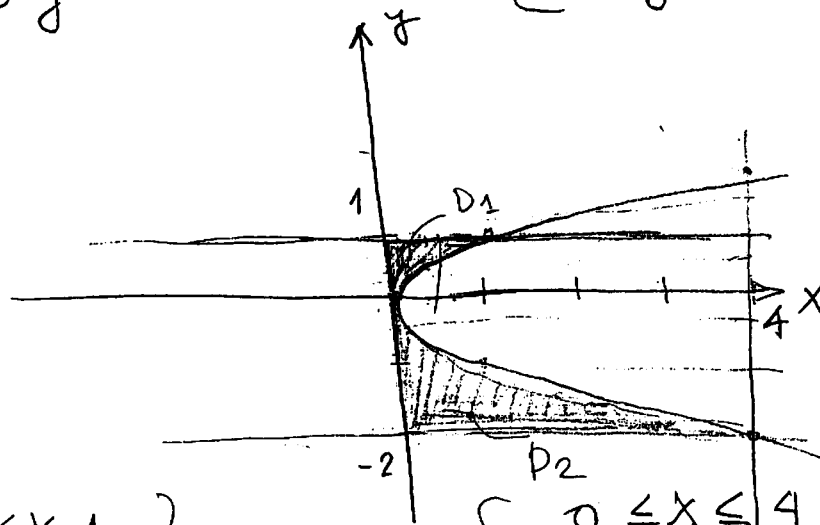
$$\boxed{-\frac{y+2}{2} = x}$$

$$= \int_0^2 dy \int_0^{-\frac{y+2}{2}} f(x,y) dx$$



6.26

$$(20.) \int_{-2}^1 dy \int_{y^2}^4 f(x,y) dx = \left\{ \begin{array}{l} -2 \leq x \leq 1 \\ y^2 \leq x \leq 4 \end{array} \right\}$$



$$y^2 \leq x$$

$$y = \sqrt{x} \quad x=1$$

$$D_1: \left\{ \begin{array}{l} 0 \leq x \leq 1 \\ \sqrt{x} \leq y \leq 1 \end{array} \right\}$$

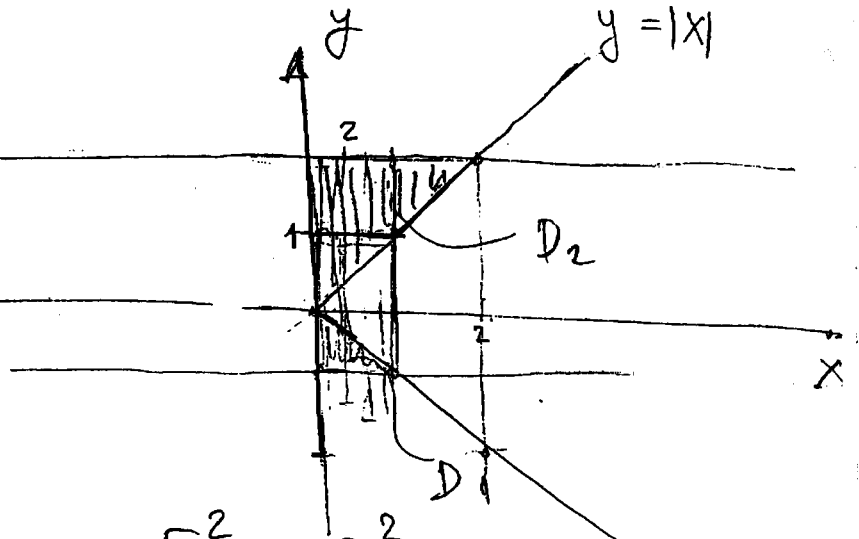
$$D_2: \left\{ \begin{array}{l} 0 \leq x \leq 4 \\ -2 \leq y \leq -\sqrt{x} \end{array} \right\}$$

$$= \int_0^1 dx \int_{\sqrt{x}}^1 f(x,y) dy + \int_0^4 dx \int_{-2}^{-\sqrt{x}} f(x,y) dy$$

$$\textcircled{8} \int_{-1}^2 dy \int_0^{|y|} f(x,y) dx = \left\{ \begin{array}{l} -1 \leq y \leq 2 \\ 0 \leq x \leq |y| \end{array} \right\} =$$

$$D_1: \left\{ \begin{array}{l} 0 \leq x \leq 1 \\ -1 \leq y \leq +x \end{array} \right\}$$

$$D_2: \left\{ \begin{array}{l} 0 \leq x \leq 2 \\ x \leq y \leq 2 \end{array} \right\}$$

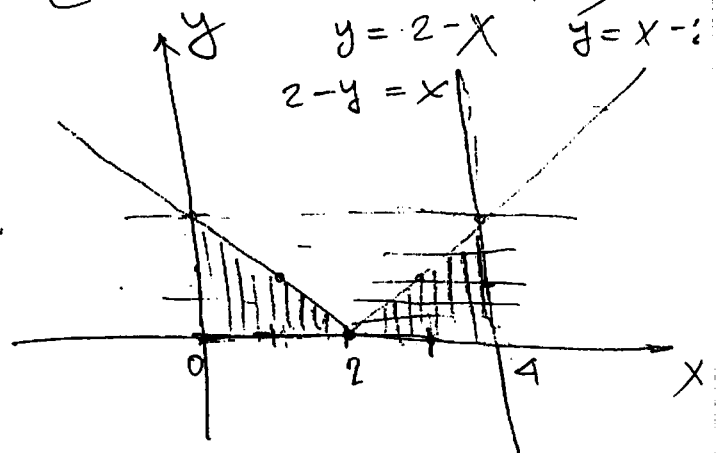


$$= \int_0^1 dx \int_{-1}^{-x} f(x,y) dy + \int_0^2 dx \int_x^2 f(x,y) dy$$

$$\textcircled{13} \int_0^4 dx \int_0^{|x-2|} f(x,y) dy = \left\{ \begin{array}{l} 0 \leq x \leq 4 \\ 0 \leq y \leq |x-2| \end{array} \right\}$$

$$D_1: \left\{ \begin{array}{l} 0 \leq x \leq 2 \\ 0 \leq y \leq 2-x \end{array} \right\}$$

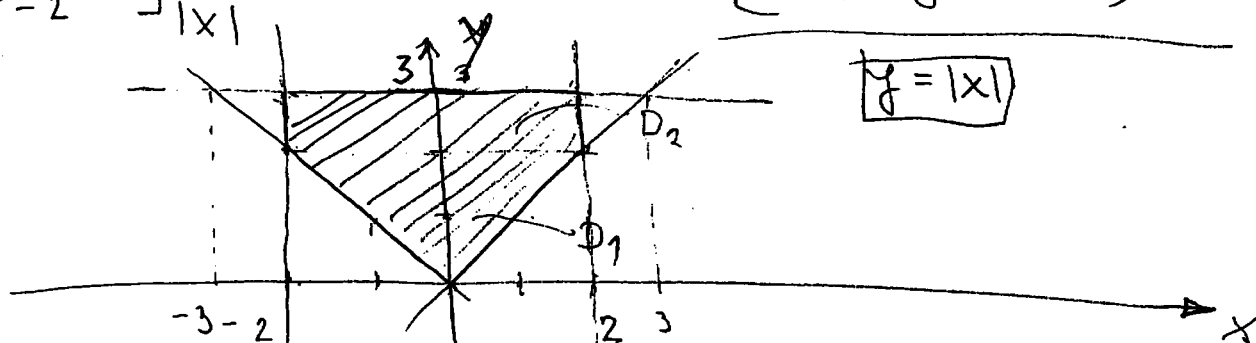
$$D_2: \left\{ \begin{array}{l} 0 \leq y \leq 2 \\ y+2 \leq x \leq 4 \end{array} \right\}$$



$$= \int_0^2 dy \int_0^{2-y} f(x,y) dx + \int_0^2 dy \int_{y+2}^4 f(x,y) dx$$

$$\begin{aligned}
 &= \int_0^1 \left(-\frac{1}{8}y^2 + \frac{1}{2}y\right) dy + \int_1^2 \left(\frac{3}{8}y^2 - \frac{3}{2}y + \frac{3}{2}\right) dy = \\
 &= -\frac{1}{8} \frac{y^3}{3} \Big|_0^1 + \frac{1}{2} \frac{y^2}{2} \Big|_0^1 + \frac{3}{8} \frac{y^3}{3} \Big|_1^2 - \frac{3}{2} \frac{y^2}{2} \Big|_1^2 + \frac{3}{2} y \Big|_1^2 = \\
 &= -\frac{1}{24} + \frac{1}{4} + \frac{1}{8}(8-1) - \frac{3}{4}(4-1) + \frac{3}{2} = \\
 &= -\frac{1}{24} + \frac{1}{4} + \frac{7}{8} - \frac{9}{4} + \frac{3}{2} = \frac{-1+6+21-54+36}{24} = \frac{8}{24} = \frac{1}{3}
 \end{aligned}$$

$$\textcircled{8} \int_{-2}^2 dx \int_{|x|}^3 (xy - y^2) dy = \left\{ \begin{array}{l} -2 \leq x \leq 2 \\ |x| \leq y \leq 3 \end{array} \right\}$$



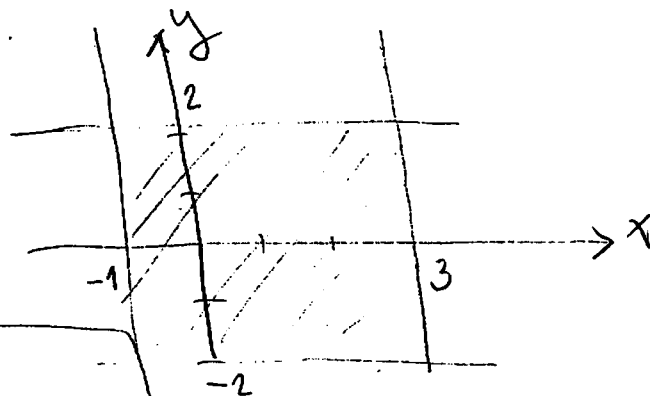
$$D_1: \left\{ \begin{array}{l} 0 \leq y \leq 2 \\ -y \leq x \leq y \end{array} \right\} \quad D_2: \left\{ \begin{array}{l} 2 \leq y \leq 3 \\ -2 \leq x \leq 2 \end{array} \right\}$$

$$\begin{aligned}
 &= \int_0^2 dy \int_{-y}^y (xy - y^2) dx + \int_2^3 dy \int_{-2}^2 (xy - y^2) dx = \\
 &= \int_0^2 dy \left(y \frac{x^2}{2} \Big|_{-y}^y - y^2 x \Big|_{-y}^y \right) + \int_2^3 dy \left(y \frac{x^2}{2} \Big|_{-2}^2 - y^2 x \Big|_{-2}^2 \right) =
 \end{aligned}$$

$$\begin{aligned}
&= \int_0^2 \left(y \left(\cancel{\frac{y^3}{2}} - \frac{y^2}{2} \right) - y^2 (y + y) \right) dy + \\
&+ \int_2^3 \left(y \left(\cancel{2 - 2} \right) - y^2 (2 + 2) \right) dy = \frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 3 \cdot 3} \\
&= -2 \int_0^2 y^3 dy - 4 \int_2^3 y^2 dy = -2 \cdot \frac{y^4}{4} \Big|_0^2 - 4 \cdot \frac{y^3}{3} \Big|_2^3 = \\
&= -8 - \frac{4}{3} (27 - 8) = -8 - \frac{76}{3} = \frac{-24 - 76}{3} = \\
&= \frac{100}{3} = \underline{\underline{\frac{100}{3}}}
\end{aligned}$$

$$B) \textcircled{3} \int_{-1}^3 dx \int_{-2}^2 f(x, y) dy = \left\{ \begin{array}{l} -1 \leq x \leq 3 \\ -2 \leq y \leq 2 \end{array} \right\}$$

$$D: \left\{ \begin{array}{l} -2 \leq y \leq 2 \\ -1 \leq x \leq 3 \end{array} \right\}$$

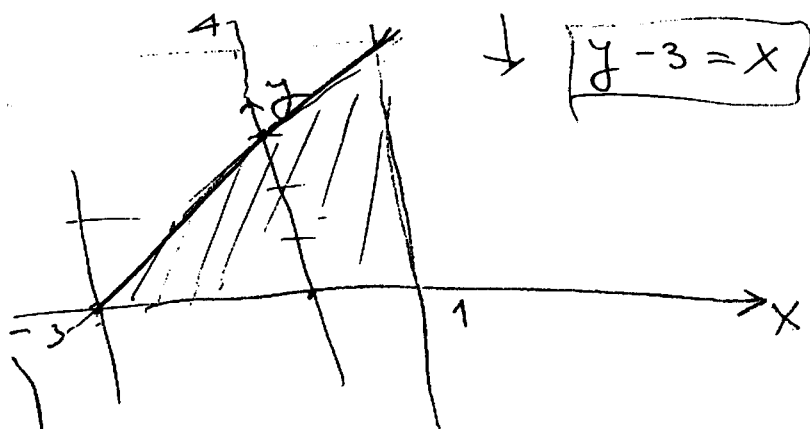


$$= \int_{-2}^2 dy \int_{-1}^3 f(x, y) dx$$

$$\textcircled{6} \int_{-3}^1 dx \int_0^{x+3} f(x,y) dy$$

$$-3 \leq x \leq 1$$

$$0 \leq y \leq x+3$$

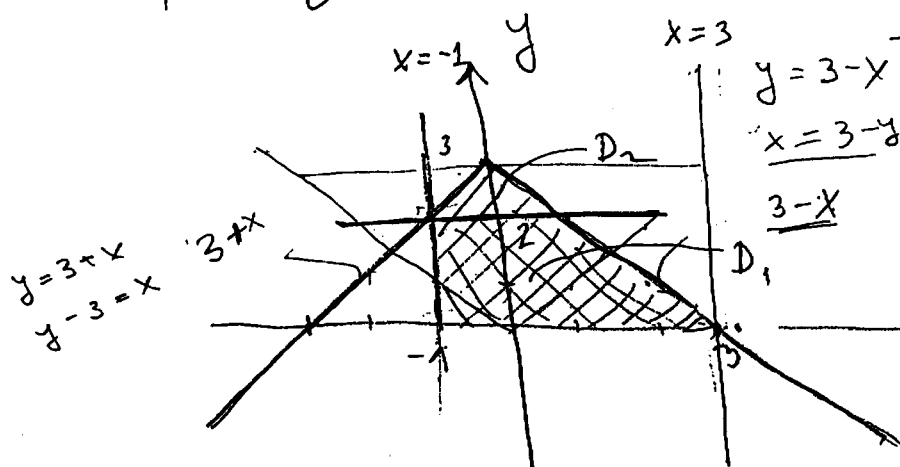


$$= \int_0^4 dy \int_{y-3}^1 f(x,y) dx$$

$$\textcircled{14} \int_{-1}^3 dx \int_0^{3-|x|} f(x,y) dy$$

$$-1 \leq x \leq 3$$

$$0 \leq y \leq 3-|x|$$



$$D_1: \begin{cases} 0 \leq y \leq 2 \\ -1 \leq x \leq 3-y \end{cases}$$

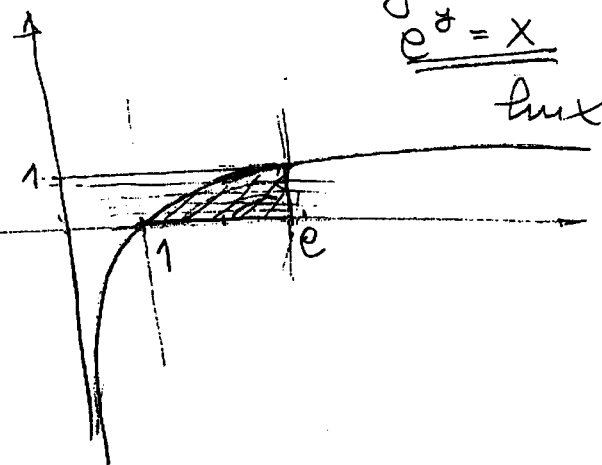
$$D_2: \begin{cases} 2 \leq y \leq 3 \\ y-3 \leq x \leq 3-y \end{cases}$$

$$= \int_0^2 dy \int_{-1}^{3-y} f(x,y) dx + \int_2^3 dy \int_{y-3}^{3-y} f(x,y) dx$$

$$(21) \int_1^e dx \int_0^{\ln x} f(x, y) dy = \begin{cases} 1 \leq x \leq e \\ 0 \leq y \leq \ln x \end{cases}$$

$y = \ln x$
 $e^y = x$

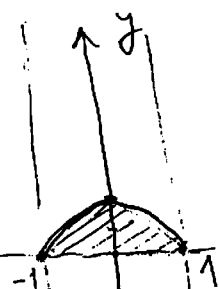
$$= \int_0^1 dy \int_{e^y}^e f(x, y) dx$$



$$(26) \int_{-1}^1 dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x, y) dy = \begin{cases} -1 \leq x \leq 1 \\ -\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2} \end{cases}$$

$y = \pm \sqrt{1-x^2}$ $y = \sqrt{1-x^2}$
 $y^2 = 1-x^2$
 $x^2 = 1-y^2$

$$= \int_0^1 dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x, y) dx$$



$$x = \pm \sqrt{1-y^2}$$

$$(29) \int_{-7}^1 dy \int_{2-\sqrt{7-6y-y^2}}^{2+\sqrt{7-6y-y^2}} f(x,y) dx$$

$$\left\{ \begin{array}{l} -7 \leq y \leq 1 \\ 2-\sqrt{7-6y-y^2} \leq x \leq 2+\sqrt{7-6y-y^2} \end{array} \right\}$$

$$(x-2)^2 + (y+3)^2 = 16$$

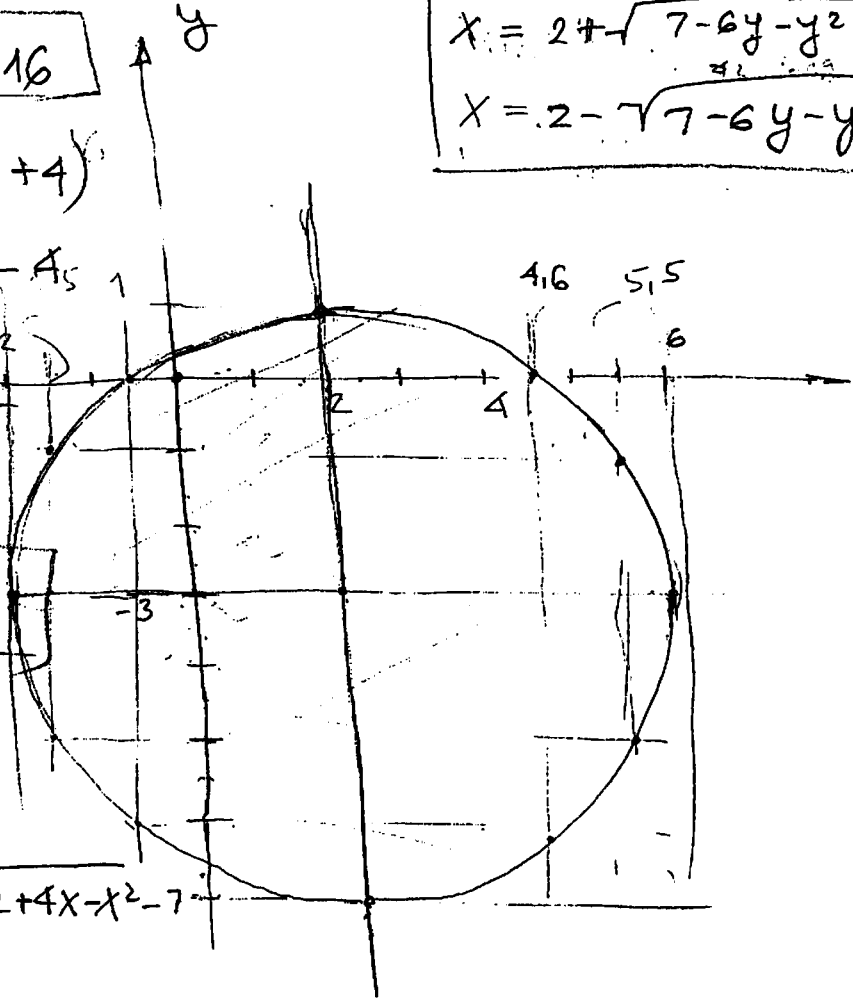
$$(y+3)^2 = 16 - (x^2 - 4x + 4)$$

$$(y+3)^2 = 16 - x^2 + 4x - 4$$

$$(y+3)^2 = -x^2 + 4x + 12$$

$$y+3 = \pm \sqrt{12+4x-x^2}$$

$$y = -3 \pm \sqrt{12+4x-x^2}$$



$$= \int_{-2}^6 dx \int_{-3-\sqrt{12+4x-x^2}}^{-3+\sqrt{12+4x-x^2}} f(x,y) dy$$

$$(33.) \int_0^{1/2} dx \int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} f(x,y) dy \Rightarrow \begin{aligned} 0 \leq x \leq \frac{1}{2} \\ 1-\sqrt{1-x^2} \leq y \leq 1+\sqrt{1-x^2} \end{aligned}$$

$$y = 1 \pm \sqrt{1-x^2}$$

$$(y-1)^2 = 1-x^2$$

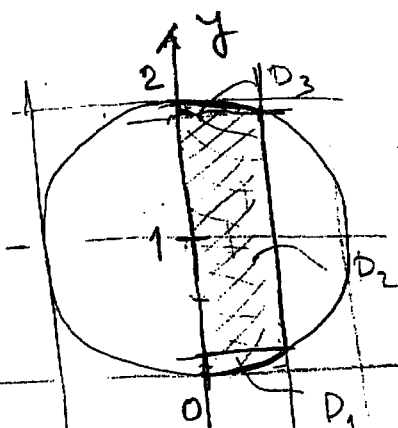
$$x^2 + (y-1)^2 = 1$$

$$x^2 = 1 - (y^2 - 2y + 1)$$

$$x^2 = 1 - y^2 + 2y - 1$$

$$x^2 = 2y - y^2$$

$$x = \pm \sqrt{2y - y^2}$$



$$= \int_0^{1-\frac{\sqrt{3}}{2}} dy \int_0^{-\sqrt{2y-y^2}} f(x,y) dx + \int_{1-\frac{\sqrt{3}}{2}}^{1+\frac{\sqrt{3}}{2}} dy \int_0^{1/2} f(x,y) dx + \int_{1+\frac{\sqrt{3}}{2}}^2 dy \int_0^{\sqrt{2y-y^2}} f(x,y) dx.$$

$$1 - \frac{1}{4} = \frac{3}{4}$$

$$1 \pm \frac{\sqrt{3}}{2} = \begin{cases} 1.866 \\ 0.134 \end{cases}$$

36.

$$\int_0^1 dx \int_{(1-x)^2/2}^{\sqrt{1-x^2}} f(x,y) dy =$$

$$0 \leq x \leq 1$$

$$\frac{(1-x)^2}{2} \leq y \leq \sqrt{1-x^2}$$

$$y^2 = 1 - x^2$$

$$x^2 + y^2 = 1$$

$$x^2 = 1 - y^2$$

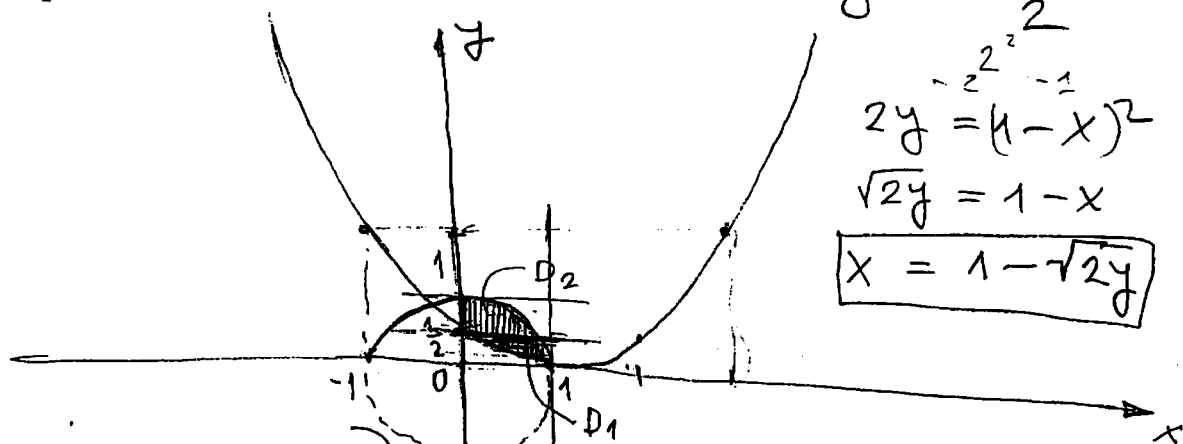
$$y = \sqrt{1-x^2}$$

$$y = \frac{(1-x)^2}{2}$$

$$2y = (1-x)^2$$

$$\sqrt{2y} = 1-x$$

$$x = 1 - \sqrt{2y}$$



$$D_1: \begin{cases} 0 \leq y < \frac{1}{2} \\ 1 - \sqrt{2y} \leq x \leq \sqrt{1-y^2} \end{cases}$$

$$D_2: \begin{cases} \frac{1}{2} \leq y \leq 1 \\ 0 \leq x \leq \sqrt{1-y^2} \end{cases}$$

$$= \int_0^{\frac{1}{2}} dy \int_{1-\sqrt{2y}}^{\sqrt{1-y^2}} f(x,y) dx + \int_{\frac{1}{2}}^1 dy \int_0^{\sqrt{1-y^2}} f(x,y) dx$$

$$(39) \int_{-2}^0 dx \int_{-\sqrt{2-(x^2/2)}}^0 f(x,y) dy =$$

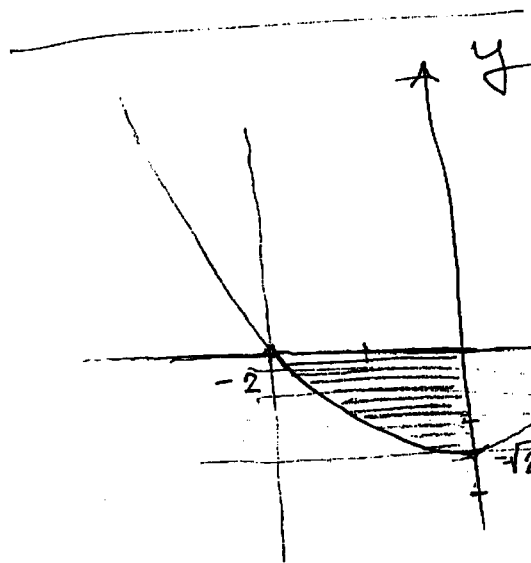
$$\begin{aligned} -2 \leq x \leq 0 \\ -\sqrt{2-(x^2/2)} \leq y \leq 0 \end{aligned}$$

$$y = -\sqrt{2-(x^2/2)}$$

$$-y = \sqrt{2-\frac{x^2}{2}}$$

$$y^2 = 2 - \frac{x^2}{2}$$

$$y^2 = \frac{4-x^2}{2}$$



$$2y^2 = 4 - x^2$$

$$2y^2 - 4 = -x^2$$

$$4 - 2y^2 = x^2$$

$$\left\{ \begin{aligned} -\sqrt{2} \leq y \leq 0 \\ -\sqrt{4-2y^2} \leq x \leq 0 \end{aligned} \right\}$$

$$\boxed{-\sqrt{4-2y^2} = x}$$

$$= \int_{-\sqrt{2}}^0 dy \int_{-\sqrt{4-2y^2}}^0 f(x,y) dx$$

$$(42) \int_1^2 dx \int_{-\sqrt{x^2-1}}^{\sqrt{x^2-1}} f(x,y) dy =$$

$$\left\{ \begin{array}{l} 1 \leq x \leq 2 \\ -\sqrt{x^2-1} \leq y \leq \sqrt{x^2-1} \end{array} \right\}$$

$$y = \pm \sqrt{x^2-1}$$

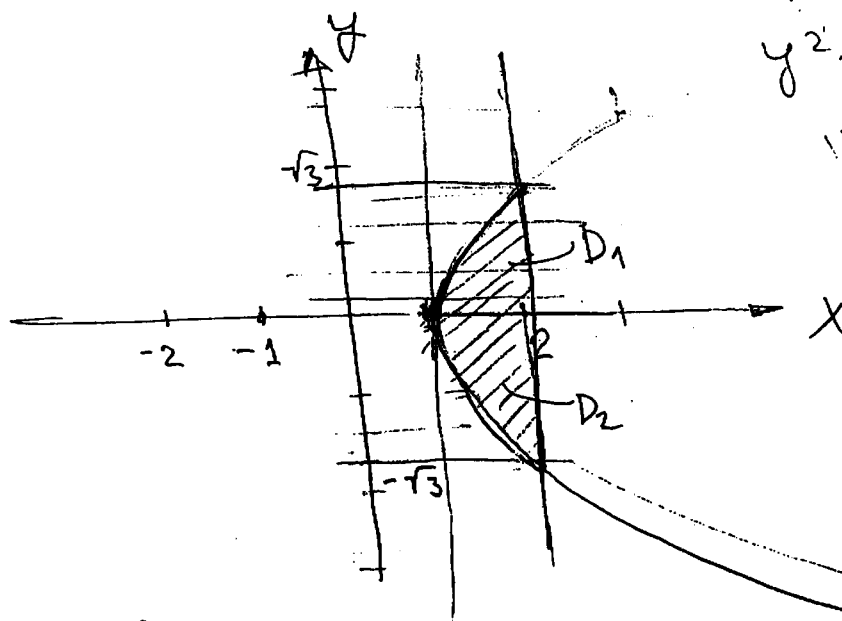
$$y^2 = x^2 - 1$$

$$y^2 - x^2 = -1$$

$$-x^2 = -y^2 - 1$$

$$x^2 = y^2 + 1$$

$$x = \pm \sqrt{y^2 + 1}$$



$$D_1: \left\{ \begin{array}{l} 0 \leq y \leq \sqrt{3} \\ \sqrt{y^2+1} \leq x \leq 2 \end{array} \right\}$$

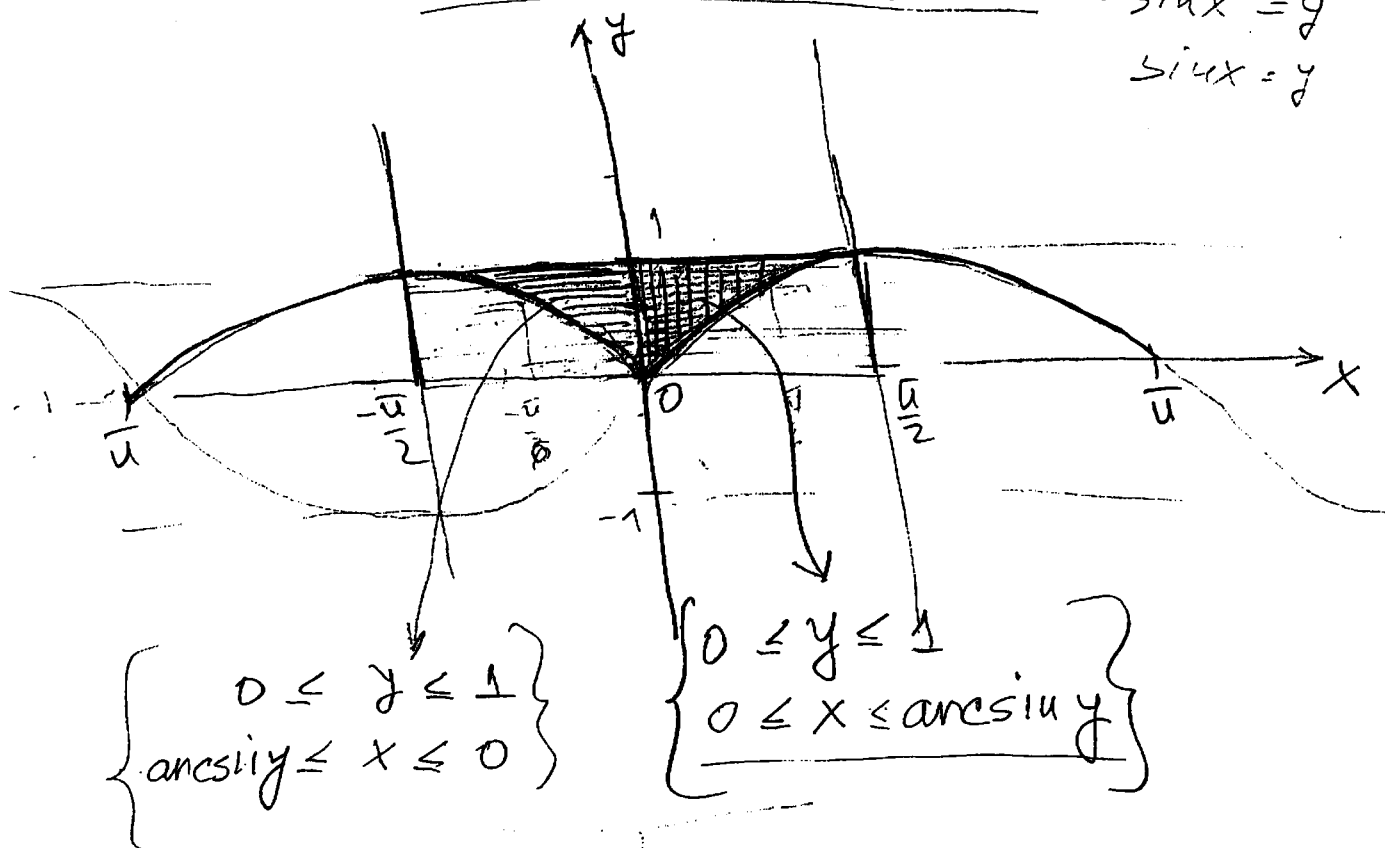
$$D_2: \left\{ \begin{array}{l} -\sqrt{3} \leq y \leq 0 \\ -\sqrt{y^2+1} \leq x \leq 2 \end{array} \right\}$$

$$= \int_0^{\sqrt{3}} dy \int_{\sqrt{y^2+1}}^2 f(x,y) dx + \int_{-\sqrt{3}}^0 dy \int_{-\sqrt{y^2+1}}^2 f(x,y) dx$$

$$(46) \int_{-\pi/2}^{\pi/2} dx \int_{|\sin x|}^1 f(x,y) dy =$$

$$\left\{ \begin{array}{l} -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \\ |\sin x| \leq y \leq 1 \end{array} \right\}$$

$$\begin{aligned} -\sin x &= y \\ \sin x &= y \end{aligned}$$



=

$$\left\{ \begin{array}{l} 0 \leq y \leq 1 \\ -\arcsin y \leq x \leq \arcsin y \end{array} \right\}$$

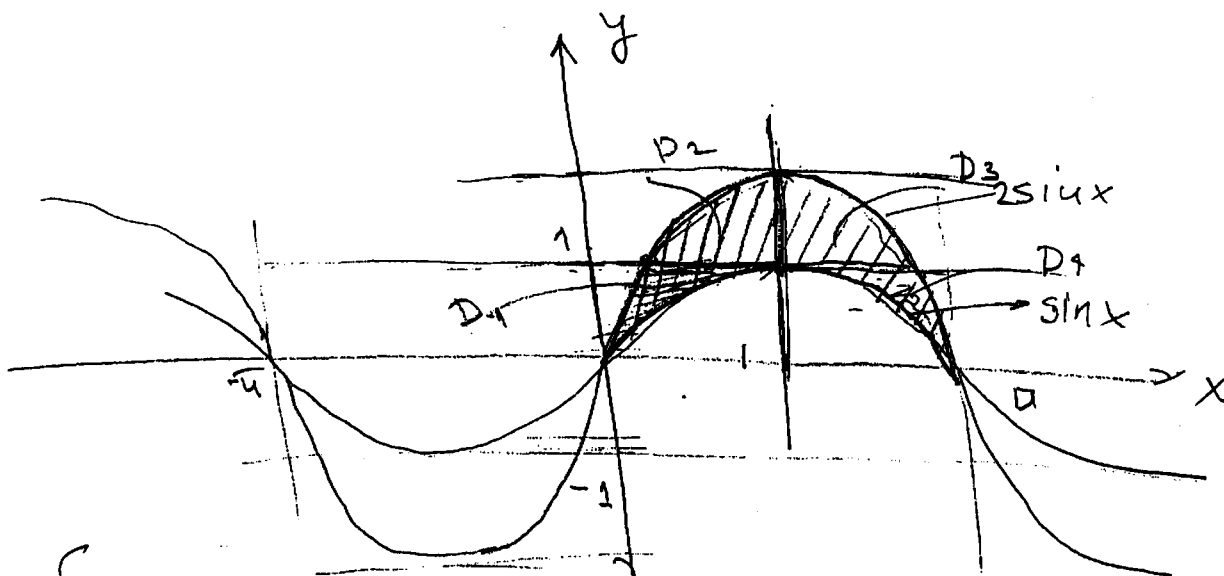
$$= \int_0^1 dy \int_{-\arcsin y}^{\arcsin y} f(x,y) dx$$

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$$(47) \int_0^{\bar{u}} dx \int_{\sin x}^{2\sin x} f(x,y) dy =$$

$$0 \leq x \leq \bar{u}$$

$$\sin x \leq y \leq 2\sin x$$

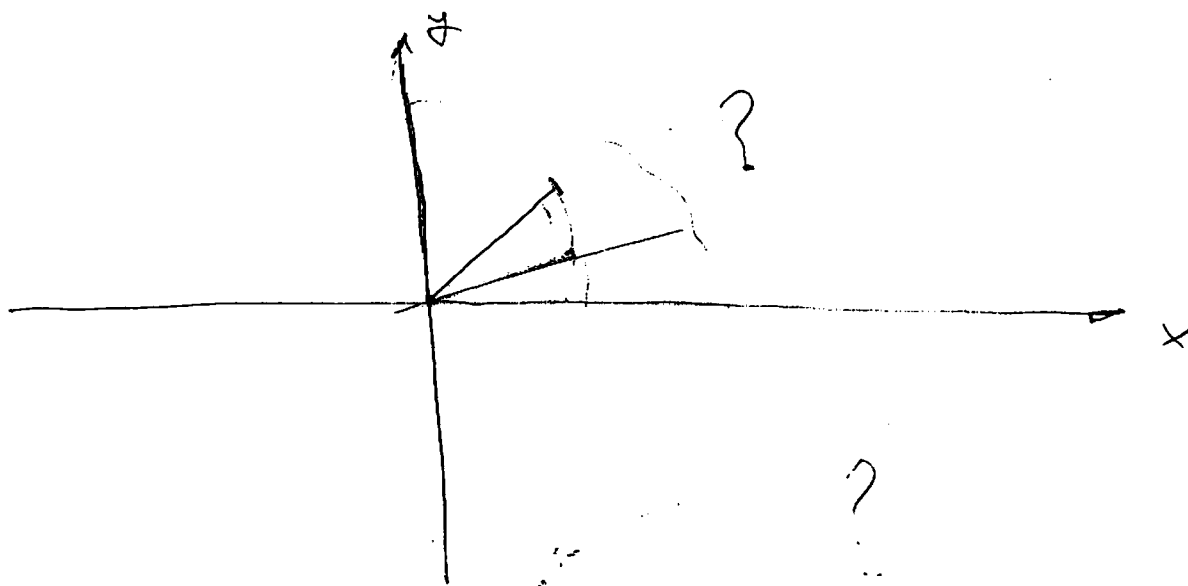


$$D_1: \begin{cases} 0 \leq y \leq 1 \\ \arcsin 2y \leq x \leq \arcsin y \end{cases}$$

$$D_2: \begin{cases} 1 \leq y \leq 2 \\ \arcsin 2y \leq x \leq \frac{\bar{u}}{2} \end{cases}$$

$$= 2 \left[\int_0^1 dy \int_{\arcsin 2y}^{\arcsin y} f(x,y) dx + \int_1^2 dy \int_{\arcsin 2y}^{\frac{\bar{u}}{2}} f(x,y) dx \right]$$

$$(52.) \int_0^{\pi/2} d\varphi \int_0^{\sqrt{\sin 2\varphi}} f(\rho, \varphi) d\rho$$



$$\left\{ \begin{array}{l} 0 \leq \varphi \leq \pi/2 \\ 0 \leq \rho \leq \sqrt{\sin 2\varphi} \end{array} \right\}$$

$$\rho^2 = \sin 2\varphi$$

$$\rho = 1 \rightarrow \frac{\pi}{4} = \varphi$$

$$\frac{\sqrt{3}}{2}$$

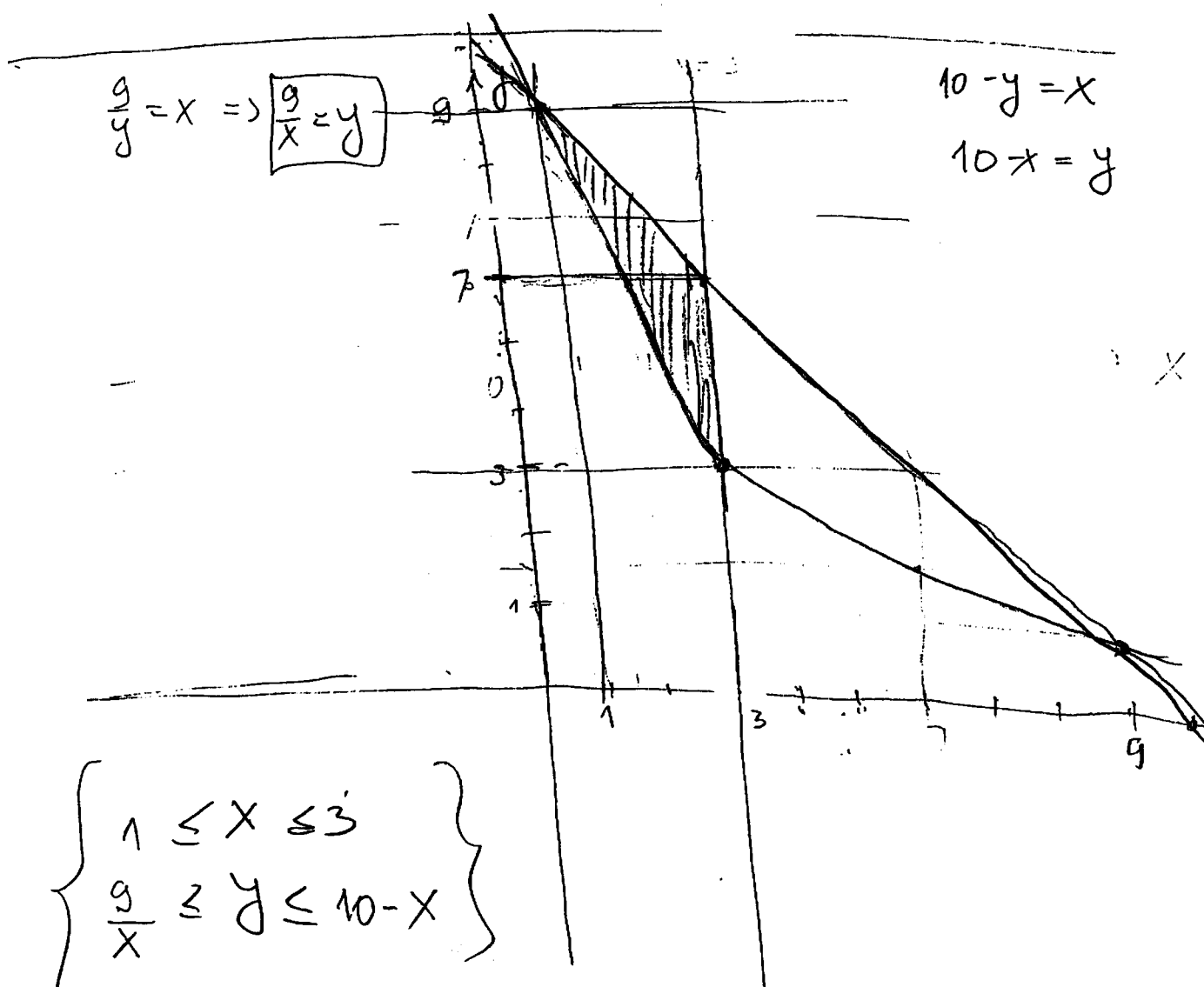
$$(55) \int_3^7 dy \int_{\frac{9}{y}}^3 f(x,y) dx + \int_7^9 dy \int_{\frac{9}{y}}^{10-y} f(x,y) dx =$$

$$3 \leq y \leq 7$$

$$\frac{9}{y} \leq x \leq 3$$

$$7 \leq y \leq 9$$

$$\frac{9}{y} \leq x \leq 10-y$$



$$\left\{ \begin{array}{l} 1 \leq x \leq 3 \\ \frac{9}{x} \leq y \leq 10-x \end{array} \right\}$$

$$= \int_1^3 dx \int_{\frac{9}{x}}^{10-x} f(x,y) dy$$