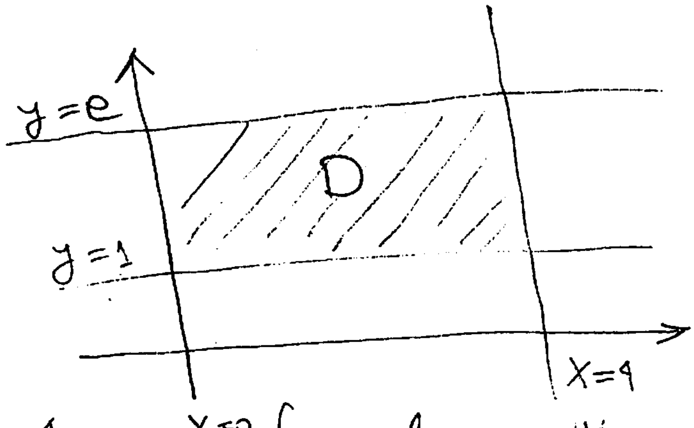
6.2. N 6.1.

1. $\iint_D x \ln y \, dx\,dy \quad , D = \{(x,y) \mid 0 \le x \le 4 \wedge 1 \le y \le e\}$



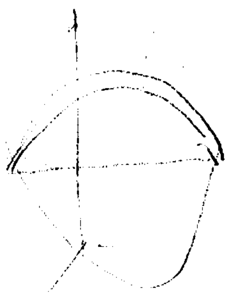
$$\int_0^4 x\,dx \int_1^e \ln y\,dy = \int_0^4 x\,dx \cdot \left\{ \begin{array}{l} u = \ln y \Rightarrow du = \frac{1}{y}dy \\ v = y \end{array} \right\}$$

$$= \int_0^4 x\,dx \cdot \left(y \ln y \Big|_1^e - \int_1^e dy \right) =$$

$$= \int_0^4 x\,dx \left((e \ln e - \ln 1) - (e-1) \right) =$$

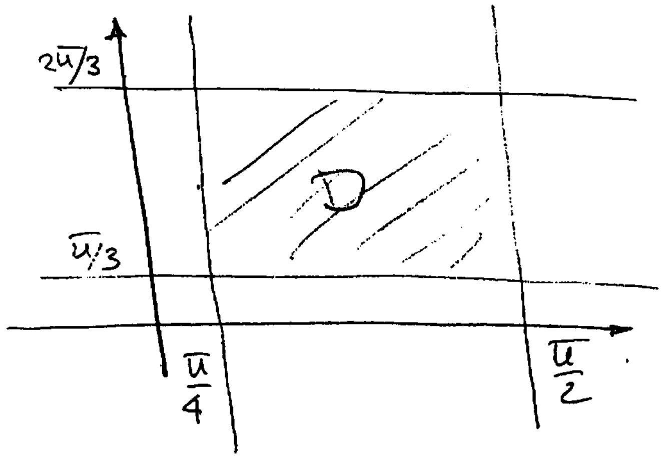
$$= \int_0^4 x\,dx \,(e - e + 1) = \int_0^4 x\,dx = \frac{x^2}{2}\Big|_0^4 =$$

$$= \frac{1}{2}(16 - 0) = \underline{\underline{8}}$$



(3.) $\iint_D (\cos^2 x + \sin^2 y)\,dx\,dy$

$D = \left\{ (x,y) \mid \frac{\pi}{4} \le x \le \frac{\pi}{2} \wedge \frac{\pi}{3} \le y \le \frac{2\pi}{3} \right\}$



$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} dx \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (\cos^2 x + \sin^2 y)\,dy =$$

$$= \int_{\pi/4}^{\pi/2} dx \left[\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \cos^2 x\,dy + \int_{\pi/3}^{\frac{2\pi}{3}} \sin^2 y\,dy \right] =$$

$$= \int_{\frac{\pi}{4}}^{\pi/2} dx \left(\cos^2 x\, y \Big|_{\pi/3}^{\frac{2\pi}{3}} + \left(-\frac{\sin y \cos y}{2} + \frac{1}{2}\int \sin y\, dy \right) \right)$$

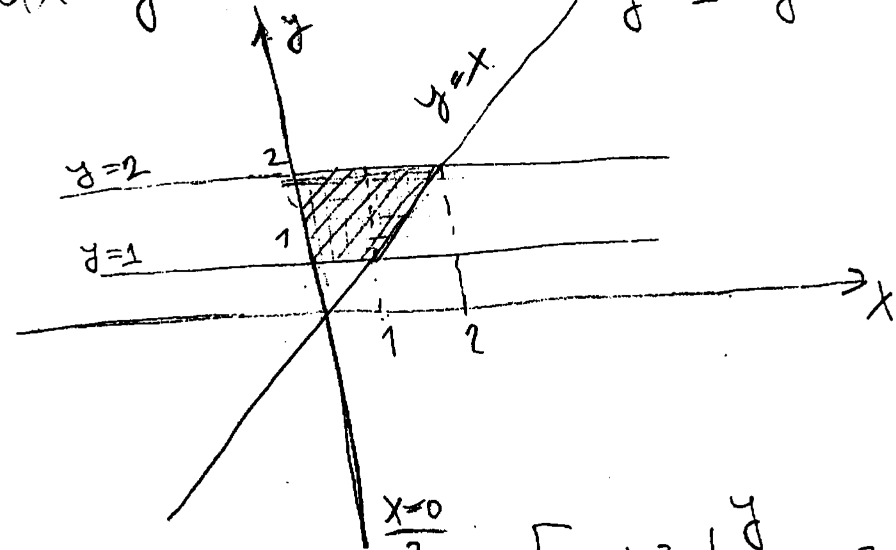
$$= \int_{\pi/4}^{\pi/2} dx \cdot \left[\cos^2 x \left(\frac{2\pi}{3} - \frac{\pi}{3} \right) - \frac{\sin 2y}{4} \Big|_{\pi/3}^{2\pi/3} + \frac{1}{2} \cos y \Big|_{\pi/3}^{2\pi/3} \right] =$$

$$= \int_{\pi/4}^{\pi/2} dx \left[\cos^2 x \cdot \left(\frac{\pi}{3} \right) - \frac{1}{4} \left(\sin \frac{4\pi}{3} - \sin \frac{2\pi}{3} \right) - \frac{1}{2} \left(\cos \frac{2\pi}{3} - \cos \frac{\pi}{3} \right) \right.$$

$$= \int_{\pi/4}^{\pi/2} dx \left[\frac{\pi}{3} \cos^2 x - \frac{1}{4} \left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \right) - \frac{1}{2} \left(-\frac{1}{2} - \frac{1}{2} \right) \right] =$$

$$= \frac{\pi}{3}\int_{\pi/4}^{\pi/2} \cos^2 x\,dx + \frac{\sqrt{3}}{4}\int_{\pi/4}^{\pi/2} dx + \frac{1}{2}\int_{\pi/4}^{\pi/2} dx = \dots$$

⑤ $\iint_D (x^2+y^2)\,dx\,dy =$ D ограничена $x=0,\ y=x$, $y=1$, $y=2$

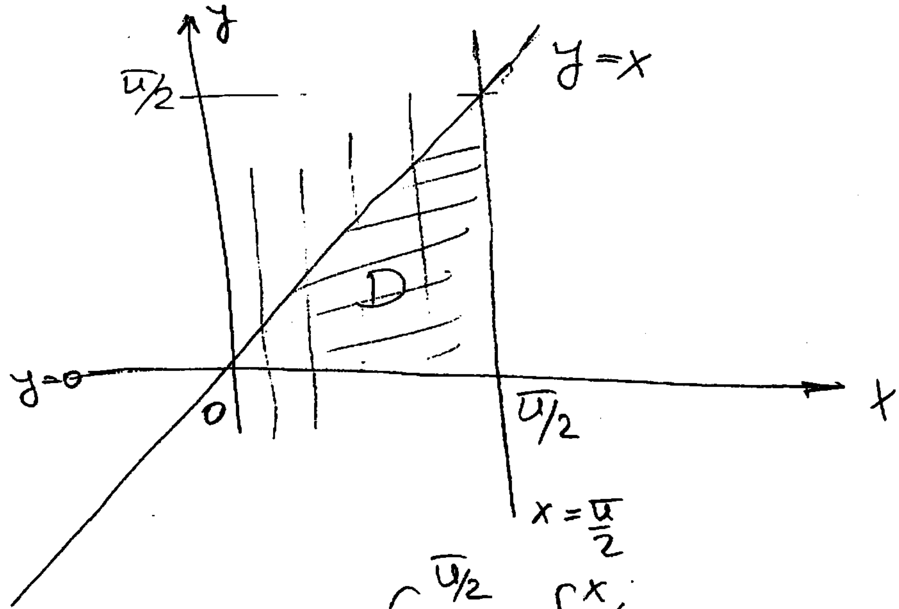


$$= \int_1^2 dy \int_0^y (x^2+y^2)\,dx = \int_1^2 dy \left[\frac{x^3}{3}\Big|_0^y + y^2 x\Big|_0^y \right] =$$

$$= \int_1^2 \left(\frac{y^3}{3} + \frac{3}{3}y^3\right) dy = \frac{4}{3}\int_1^2 y^3\,dy = \frac{\cancel{4}}{3}\,\frac{y^4}{\cancel{4}}\Big|_1^2 =$$

$$= \frac{1}{3}(16-1) = \underline{\underline{\frac{15}{3}}}$$

7. $\iint\limits_D \sin(x+y)\,dx\,dy$ D ограничена линијама $y=0,\ x=\frac{\pi}{2},\ x=y$



$$\int_0^{\pi/2} dx \int_0^x \sin(x+y)\,dy = \int_0^{\pi/2} dx \int_0^x (\sin x\cos y + \cos x\sin y)\,dy$$

$$= \int_0^{\pi/2} dx \left[\int_0^x \sin x\cos y\,dy + \int_0^x \cos x\sin y\,dy\right] =$$

$$= \int_0^{\pi/2} \left[\sin x\sin y\Big|_0^x - \cos x\cos y\Big|_0^x\right] dx =$$

$$= \int_0^{\pi/2} \left[\sin x(\sin x - \sin 0) - \cos x(\cos x - \cancel{\cos 0}^{\,1})\right] dx =$$

$$= \int_0^{\pi/2} \left[\sin^2 x - \cos^2 x + \cos x\right] dx =$$

$$= \int_0^{\pi/2} \left[\sin^2 x - (1 - \sin^2 x) + \cos x\right] dx =$$

$$= \int_0^{\pi/2} \left[2\sin^2 x + \cos x - 1\right] dx = \quad 4$$

$$= 2\int_0^{\pi/2} \sin^2 x\,dx + \int_0^{\pi/2} \cos x\,dx + \int_0^{\pi/2} dx =$$

$$= \cancel{2}\left(\frac{\cos x \sin x}{-\cancel{2}}\Big|_0^{\pi/2} + \frac{1}{2}\int_0^{\pi/2} \sin x\,dx \right) + \sin x\Big|_0^{\pi/2} - x\Big|_0^{\pi/2} =$$

$$= \left(\cos\frac{\pi}{2}\sin\frac{\pi}{2} - \cos 0 \sin 0\right) - \frac{1}{2}\left(\cos\frac{\pi}{2} - \cos 0\right) + \left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$-\left(\frac{\pi}{2} - 0\right) = \frac{1}{2} + \frac{2}{2} - \frac{\pi}{2} = \boxed{\frac{3-2\pi}{2}}$$

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10. $\iint_D (2x+3y+1)\,dx\,dy$ D је ограничена са тјеменима:

$A(-1,-1),\ B(2,0),\ C(1,1)$

$x=2 \quad y=0$ $\quad -2\alpha = \beta$

$x=1 \quad y=1$

$0 = 2\alpha + \beta$

$1 = \alpha + \beta$

$1 = \alpha - 2\alpha$

$1 = -\alpha$

$\boxed{\alpha = -1}$

$\boxed{\beta = 2}$

$x=2 \quad y=0$

$x=-1 \quad y=-1$

$y = \alpha x + \beta$

$0 = 2\alpha + \beta$

$-1 = -\alpha + \beta$

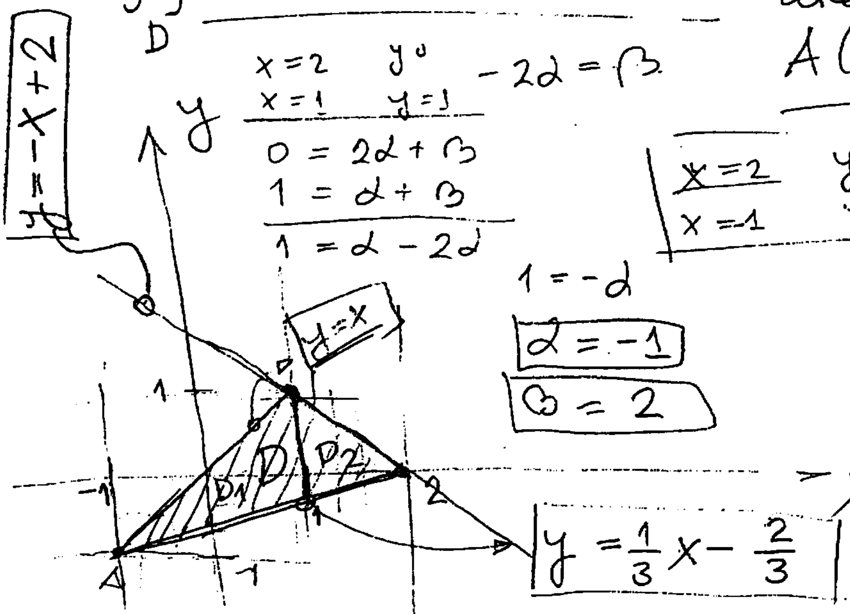
$-2\alpha = \beta$

$-1 = -\alpha - 2\alpha \ /(-1)$

$1 = 3\alpha$

$\boxed{\alpha = \frac{1}{3}}$

$\boxed{\beta = -\frac{2}{3}}$



$3y = x - 2$

$3y = x - 2$

$\boxed{x = 3y + 2}$

$\boxed{x = -y + 2}$

$x = y$

$$I = I_1 + I_2$$

$$\iint_D = \iint_{D1} + \iint_{D2}$$

S

$$I_1 = \int_{-1}^{1} dx \int_{\frac{x-2}{3}}^{x} (2x+3y+1)\,dy =$$

$$\frac{9x^2 - x^2 + 2x - 4}{4 \quad 9} \qquad \frac{\cancel{8}x^2 + \cancel{2}x - \cancel{4}^2}{\cancel{9}\ 3}$$

$$= \int_{-1}^{1} dx \left[2xy \Big|_{\frac{x-2}{3}}^{x} + 3\frac{y^2}{2} \Big|_{\frac{x-2}{3}}^{x} + y \Big|_{\frac{x-2}{3}}^{x} \right] = \qquad \frac{3x-x+2}{3} \quad \frac{2x+2}{3}$$

$$\frac{3x-x+2}{3} \cdot 2x \qquad 2x+2$$

$$= \int_{-1}^{1} dx \left[2x\left[x - \frac{x-2}{3}\right] + \frac{3}{\cancel{2}}\left[x^2 - \frac{(x-2)^2}{9}\right] + \left[x - \frac{x-2}{3}\right]\right] =$$

$$= \int_{-1}^{1} \left[\frac{4}{3}(x+1)x + \frac{1}{3}(4x^2+x-2) - \frac{2}{3}(x+1)\right] dx =$$

$$= \frac{4}{3}\int_{-1}^{1} (x+1)x\,dx + \frac{1}{3}\int_{-1}^{1} (4x^2+x-2)\,dx - \frac{2}{3}\int_{-1}^{1} (x+1)\,dx =$$

$$= \frac{4}{3}\left(\frac{x^3}{3}\Big|_{-1}^{1} + \frac{x^2}{2}\Big|_{-1}^{1}\right) + \frac{1}{3}\left(4\frac{x^3}{3}\Big|_{-1}^{1} + \frac{x^2}{2}\Big|_{-1}^{1} - 2x\Big|_{-1}^{1}\right) -$$

$$-\frac{2}{3}\left(\frac{x^2}{2}\Big|_{-1}^{1} + x\Big|_{-1}^{1}\right) =$$

$$= \frac{4}{3}\left(\frac{1}{3} + \frac{1}{3}^{\frac{2}{3}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}}\right) + \frac{1}{3}\left(\frac{4}{3} + \frac{4}{3}^{\frac{8}{3}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} - 2(1+1)^{2}_{-4}\right) -$$

$$-\frac{2}{3}\left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} + 1 + 1\right) = \frac{8}{9} + \frac{8}{9} - \frac{4}{3} - \frac{4}{3} = -\frac{8}{3}$$

$$= \frac{16-24}{9} = -\frac{8}{9}$$

$$36-18x+9x^2-x^2+2x-4$$
$$\frac{9(4-2x+x^2)-(x^2-2x+4)}{9}$$
$$\frac{8x^2-16x+32}{9}$$

$$I_2=\int_1^2 dx\int_{\frac{x-2}{3}}^{2-x}(2x+3y+1)\,dy=$$

$$=\int_1^2 dx\left[2x\cdot y\Big|_{\frac{x-2}{3}}^{2-x}+\frac{3}{2}y^2\Big|_{\frac{x-2}{3}}^{2-x}+y\Big|_{\frac{x-2}{3}}^{2-x}\right]$$

$$\frac{4}{9}(x^2-2x+4)$$
$$\frac{6-3x-x+2}{3} \quad \frac{8-4x}{3}$$

$$\frac{6-3x-x+2}{3}=$$

$$=\int_1^2\left[2x\left((2-x)-\frac{x-2}{3}\right)+\frac{3}{2}\left((2-x)^2-\frac{(x-2)^2}{9}\right)+(2-x)-\frac{x-2}{3}\right]dx=$$

$$4\,\frac{8-4x}{3}\,2x$$

$$=\int_1^2\left[\frac{8}{3}(2x-x^2)+\frac{4}{3}(x^2-2x+4)+\frac{4}{3}(2-x)\right]dx=$$

$$=\frac{8}{3}\left(\cancel{2}\frac{x^2}{\cancel{2}}\Big|_1^2-\frac{x^3}{3}\Big|_1^2\right)+\frac{4}{3}\left(\frac{x^3}{3}\Big|_1^2-\cancel{2}\frac{x^2}{\cancel{2}}\Big|_1^2+4x\Big|_1^2\right)+$$
$$+\frac{4}{3}\left(2x\Big|_1^2-\frac{x^2}{2}\Big|_1^2\right)=\frac{1}{3}-3+4=\frac{1-9+12}{3}=\frac{4}{3}$$

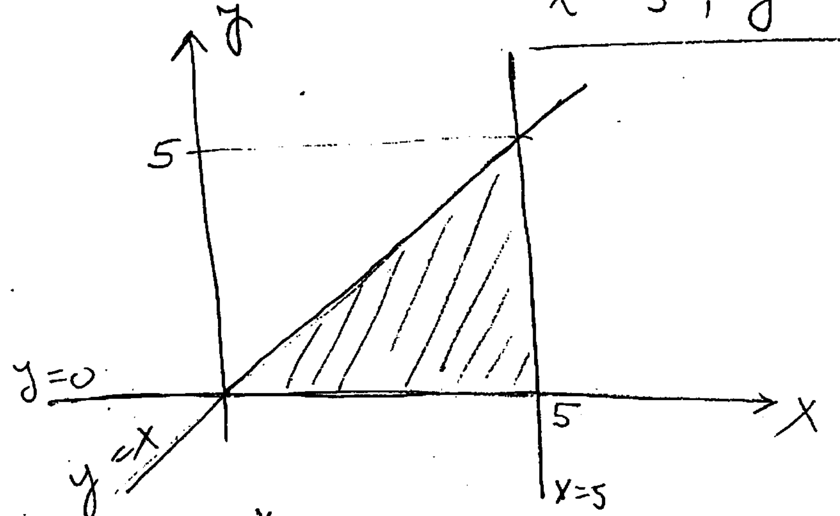
$$\frac{9-8+12}{3} \quad 3$$

$$=\frac{8}{3}\left(4-1-\left(\frac{8}{3}+\frac{1}{3}\right)\right)+\frac{4}{3}\left(\frac{2}{3}-\frac{1}{3}-(4-1)+4(2-1)\right)+$$
$$+\frac{4}{3}\left(2(2-1)-\left(\frac{4}{2}-\frac{1}{2}\right)\right)=\quad 2-\frac{3}{2}=\frac{4-3}{2}=\frac{1}{2}$$

$$=\frac{16}{9}+\frac{16}{9}+\frac{2}{3}=\frac{16+16+6}{9}=\frac{38}{9}$$

$$\boxed{I=-\frac{8}{9}+\frac{38}{9}=\frac{30}{9}}$$

(12.) $\iint_D (x+2y)\,dx\,dy$, D ограничена линијама
$x=5,\ y=0,\ y=x$

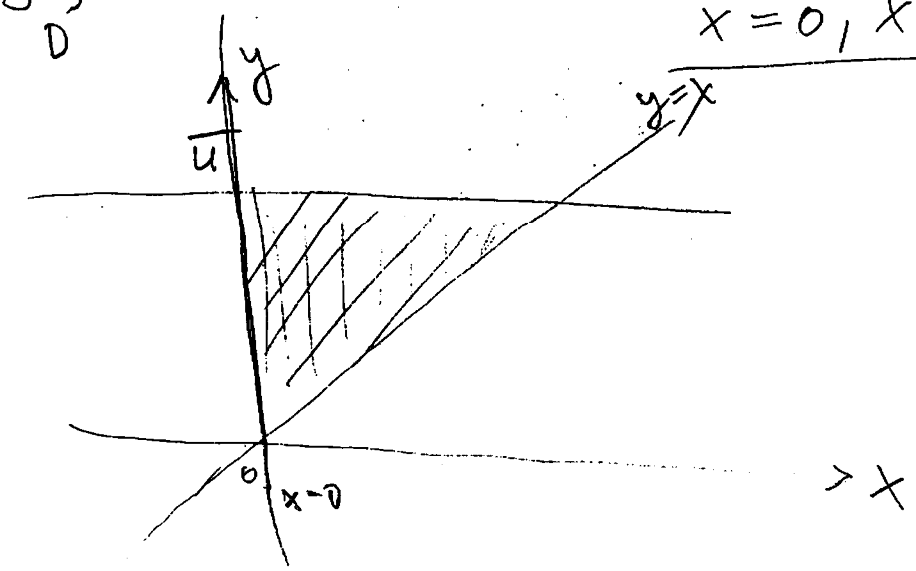


$$I=\int_0^5 dx\int_0^x (x+2y)\,dy=\int_0^5 dx\left[xy\Big|_0^x+\cancel{2}\frac{y^2}{\cancel{2}}\Big|_0^x\right]$$

$$=\int_0^5 dx\,(x^2+x^2)=\int_0^5 2x^2\,dx$$

$$I=\frac{2}{3}x^3\Big|_0^5=\frac{2}{3}(125-0)=\frac{250}{3}$$

(14.) $\iint_D \cos(x+y)\,dx\,dy$, D ограничена линијама
$x=0,\ x=y,\ y=\pi$



$$I = \int_0^{\pi} dy \int_0^{y} \cos(x+y)\,dx =$$

$$= \int_0^{\pi} dy \int_0^{y} (\cos x \cos y - \sin x \sin y)\,dx =$$

$$= \int_0^{\pi} dy \left[\cos y \cdot \sin x \Big|_0^{y} + \sin y \cdot \cos x \Big|_0^{y}\right] =$$

$$= \int_0^{\pi} \Big[(\cos y \sin y - \cos y \cdot \overset{0}{\cancel{\sin 0}}) + (\sin y \cos y - \sin y \cos 0)\Big]\,dy$$

$$\boxed{\sin 2x} = 2\sin x \cos x$$

$$I = \int_0^{\pi} (\underbrace{\cos y \sin y + \sin y \cos y} - \sin y)\,dy$$

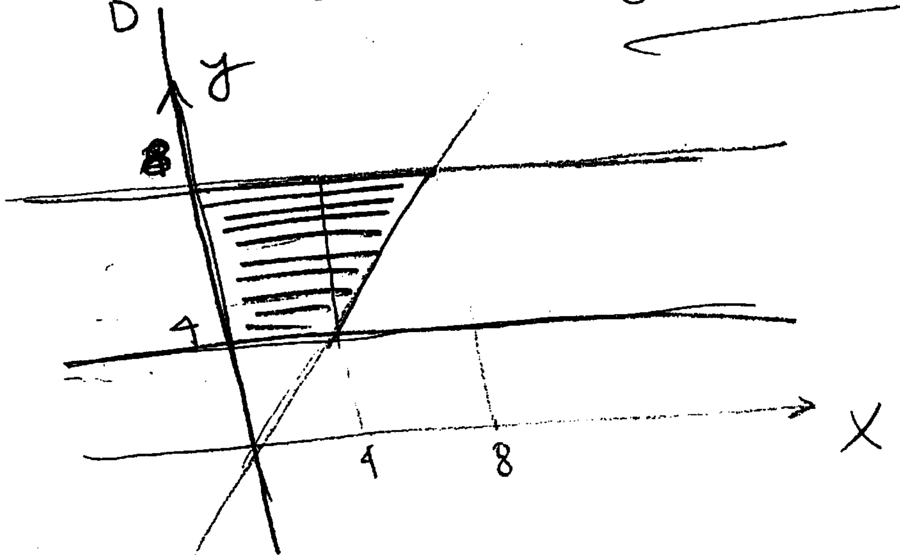
$$I = \int_0^{\pi} \sin 2y\,dy - \int_0^{\pi} \sin y\,dy =$$

$$= -\frac{1}{2}\cos 2y \Big|_0^{\pi} + \cos y \Big|_0^{\pi} = -1$$

$$I = -\frac{1}{2}(\overset{1}{\cancel{\cos 2\pi}} - \overset{1}{\cancel{\cos 0}}) + (\overset{-1}{\cancel{\cos \pi}} - \overset{1}{\cancel{\cos 0}})$$

$$\boxed{I = -2}\ ?$$

(16) $\iint\limits_D \frac{y^3}{x^2+y^2}\,dx\,dy$, D ограничено линијама $y=4$, $y=8$, $y=x$, $x=0$



$\frac{1}{x^2+a^2}$

$$I=\int_4^8 dy\int_0^y \frac{y^3}{x^2+y^2}\,dx=\int_4^8 y^3\,dy\int_0^y\frac{1}{x^2+y^2}\,dx=$$

$$=\int_4^8 y^3\,dy\cdot\left(\frac{1}{y}\operatorname{arctg} x\Big|_0^y\right)=$$

$$I=\int_4^8 y^3\,dy\cdot\left(\frac{1}{y}\left(\operatorname{arctg} y-\underbrace{\cancel{\operatorname{arctg} 0}}_{0}\right)\right)=$$

$$=\int_4^8 y^2\operatorname{arctg} y\,dy=\begin{cases}u=\operatorname{arctg} y\to du=\frac{1}{1+y^2}\,dy\\ v=\frac{y^3}{3}\end{cases}$$

$$=\frac{y^3}{3}\operatorname{arctg} y\Big|_4^8-\int_4^8\frac{1}{1+y^2}\cdot\frac{y^3}{3}\,dy$$

$$\frac{1}{3}\int_4^8 \frac{y^3}{1+y^2}\,dy = \frac{1}{3}\int_4^8 \frac{(1+y^2)y - y}{1+y^2}\,dy =$$

$$= \frac{1}{3}\frac{y^2}{2}\Big|_4^8 - \frac{1}{3}\int_4^8 \frac{y}{1+y^2}\,dy = \left\{\begin{array}{l} 1+y^2=t \\ 2y\,dy = dt \end{array}\right.$$

$$= \frac{1}{3}\frac{y^2}{2}\Big|_4^8 - \frac{1}{3}\cdot\frac{1}{2}\ln|1+y^2|\Big|_4^8 =$$

$$= \frac{1}{6}(64-16) - \frac{1}{6}(\ln 65 - \ln 17)$$

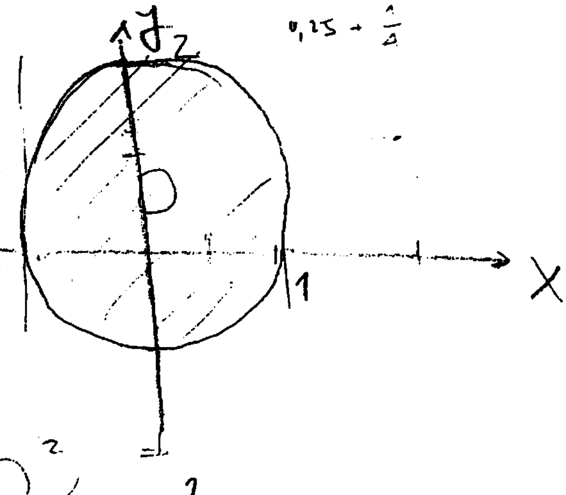
$$= \boxed{8 - \frac{1}{6}\ln\frac{65}{17}}$$ ✓

(18.) $\iint_D xy\,dx\,dy$, $D = \{(x,y) \mid 4x^2+y^2 \leq 4\}$

$\frac{x^2}{1} + \frac{y^2}{4} \leq 1$

$y^2 = 4-4x^2$

$$\boxed{\begin{array}{l} y_1 = \sqrt{4-4x^2} \\ y_2 = -\sqrt{4-4x^2} \end{array}}$$

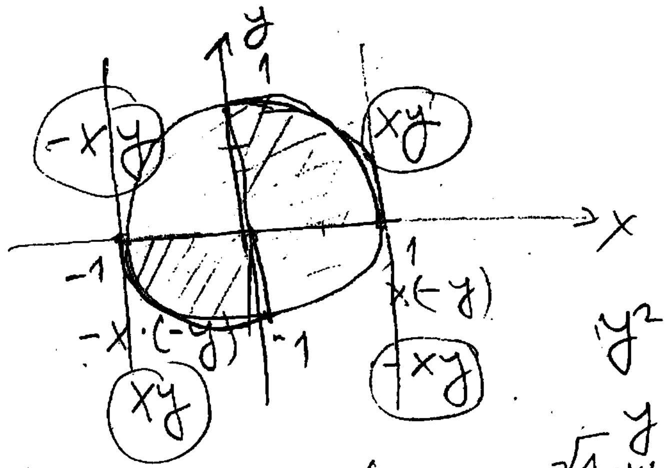


$$\int_{-1}^{1} dx \int_{-\sqrt{4-4x^2}}^{\sqrt{4-4x^2}} xy\,dy =$$

$$= \int_{-1}^{1} x\,dx \cdot \frac{y^2}{2}\Big|_{-\sqrt{4-4x^2}}^{\sqrt{4-4x^2}} = \frac{1}{2}\int_{-1}^{1} x(4-\cancel{4x^2}-4+4x^2)\,dx$$

?

(20.) $\iint_D |xy|\,dx\,dy$, $D = \{(x,y) \mid x^2+y^2 \le 1\}$



$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$

$$I = \int_{-1}^{1} dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} |xy|\,dy = 2\int_{-1}^{1} dx \cdot 2\int_{0}^{\sqrt{1-x^2}} \; ?$$

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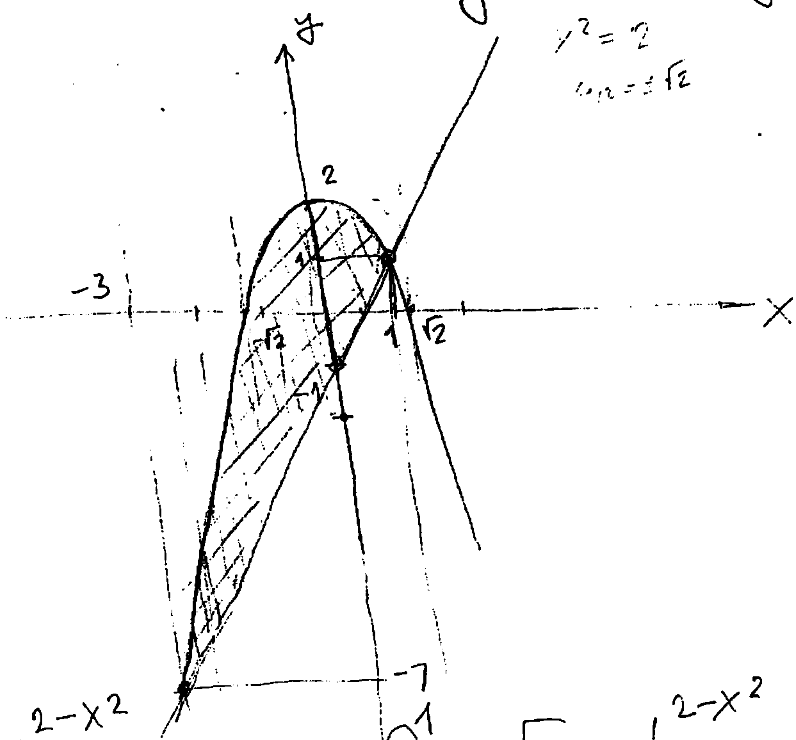
(22) $\iint_D (x-y)\,dx\,dy$, D ограничена линијама

$y = 2-x^2$, $y = 2x-1$

$x^2 = 2$ $\quad 2x = 1$

$x_{1,2} = \pm\sqrt{2}$ $\quad x = \frac{1}{2}$



$2 - x^2 = 2x - 1$

$2 + 1 = 2x + x^2$

$x^2 + 2x - 3 = 0$

$x_{1,2} = \frac{-2 \pm \sqrt{4+12}}{2}$

$x_{1,2} = \frac{-2 \pm 4}{2}$ $\;\; 1, \; -3$

$$I = \int_{-3}^{1} dx \int_{2x-1}^{2-x^2} (x-y)\,dy = \int_{-3}^{1} dx \left[xy \Big|_{2x-1}^{2-x^2} - \frac{y^2}{2}\Big|_{2x-1}^{2-x^2} \right] =$$

$$I = \int_{-\sqrt{R}}^{\sqrt{R}} dx \int_{-\sqrt{R-x^2}}^{\sqrt{R-x^2}} y^2 \sqrt{R^2-x^2}\, dy =$$

$$= \int_{-\sqrt{R}}^{\sqrt{R}} \sqrt{R^2-x^2}\, dx \int_{-\sqrt{R-x^2}}^{\sqrt{R-x^2}} y^2 dy \Rightarrow$$

?

$$I = \int_{-\sqrt{R}}^{\sqrt{R}} \sqrt{R^2-x^2}\, dx \left(\frac{y^3}{3} \Big|_{-\sqrt{R-x^2}}^{\sqrt{R-x^2}} \right) =$$

$$= \frac{1}{3} \int_{-\sqrt{R}}^{\sqrt{R}} \left(\sqrt{(R-x^2)^3} + \sqrt{(R-x^2)^3} \right) \sqrt{R^2-x^2}\, dx =$$

$$(R-x^2)(R^2-x^2)$$
$$\underbrace{R^3 - R^2x^2 - x^2R + x}$$

$$= \frac{1}{3} \int_{-\sqrt{R}}^{\sqrt{R}} 2\sqrt{(R-x^2)^3} \sqrt{R^2-x^2}\, dx = \frac{2}{3} \int_{-\sqrt{R}}^{\sqrt{R}} (R-x^2)\sqrt{R-x^2} \cdot \sqrt{R^2-x}$$

$$I=\int_1^2 dx\int_{\frac1x}^{\sqrt{x}} y\ln x\,dy = \int_1^2 \ln x\,dx\int_{\frac1x}^{\sqrt{x}} y\,dy =$$

$$I=\int_1^2 \ln x\,dx\left[\frac{y^2}{2}\Big|_{\frac1x}^{\sqrt{x}}\right] = \frac12\int_1^2 \left(x-\frac{1}{x^2}\right)\ln x\,dx =$$

$$\frac{x^{-2+1}}{-2+1}$$

$$=\frac12\int_1^2 x\ln x\,dx - \frac12\int_1^2\frac{\ln x\,dx}{x^2} =$$

$$=\frac12\cdot\left\{\begin{matrix} u=\ln x \to du=\frac1x dx\\ \frac{x^2}{2}=v\end{matrix}\right\} - \frac12\cdot\left\{\begin{matrix} u=\ln x \to du=\frac1x dx \\ v=-\frac1x\end{matrix}\right\} =$$

$$=\frac12\left[\frac{x^2}{2}\ln x\Big|_1^2 - \frac12\int_1^2 x\,dx\right] - \frac12\left[-\frac1x\ln x\Big|_1^2 + \int\frac{1}{x^2}dx\right]=$$

$$=\frac{x^2}{4}\ln x\Big|_1^2 - \frac14\cdot\frac{x^2}{2}\Big|_1^2 + \frac{1}{2x}\ln x\Big|_1^2 + \frac12\frac1x\Big|_1^2 = \quad -\frac12$$

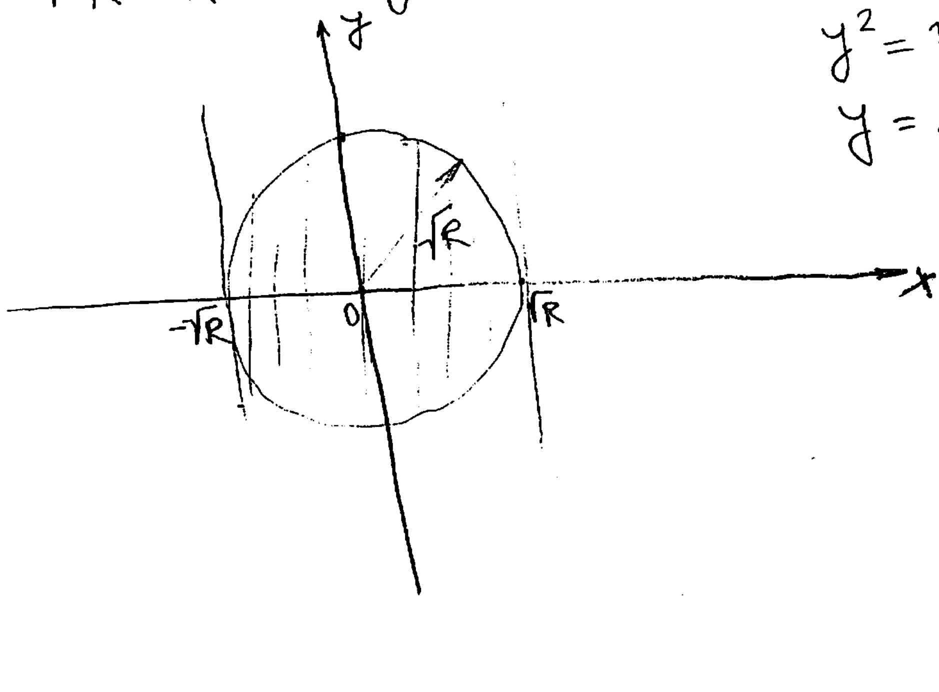
$$=\frac14(4\ln 2-\cancel{\ln 1}^{0}) - \frac18(4-1) + \frac12\left(\frac{\ln 2}{2} - \cancel{\ln 1}^{0}\right) + \frac12\left(\frac12 - 1\right) =$$

$$=\ln 2 - \frac38 + \frac{\ln 2}{4} - \frac14 = \frac54\ln 2 - \frac58$$

(26) $\iint_D y^4\sqrt{R^2-x^2}\,dx\,dy$, $D=\{(x,y)\mid x^2+y^2\le R\}$

$y^2=R-x^2$

$y=\pm\sqrt{R-x^2}$



$$I=\int_{-3}^{1}\left[x\left(2-x^2-2x+1\right)-\frac{1}{2}\left(\underline{(2-x^2)^2-(2x-1)^2}_{\,4-2x^2+x^4-(4x^2-4x+1)}\right)\right]dx=$$

$$I=\int_{-3}^{1}\left(-x^3-2x^2+3x-\frac{1}{2}\left(\underbrace{4-2x^2+x^4-4x^2+4x-1}_{x^4-6x^2+4x+3}\right)\right)dx$$

$$I=\int_{-3}^{1}\left(-x^3-2x^2+3x-\frac{x^4}{2}+3x^2-2x+\frac{3}{2}\right)dx$$

$$I=\int_{-3}^{1}\left(-\frac{x^4}{2}-x^3+x^2+x+\frac{3}{2}\right)dx=$$

$$=-\frac{1}{2}\frac{x^5}{5}\Big|_{-3}^{1}-\frac{x^4}{4}\Big|_{-3}^{1}+\frac{x^3}{3}\Big|_{-3}^{1}+\frac{x^2}{2}\Big|_{-3}^{1}+\frac{3}{2}x\Big|_{-3}^{1}=$$

$$=-\frac{1}{10}(1+243)-\frac{1}{4}(1-81)+\frac{1}{3}(1+27)+\frac{1}{2}(1-9)+$$

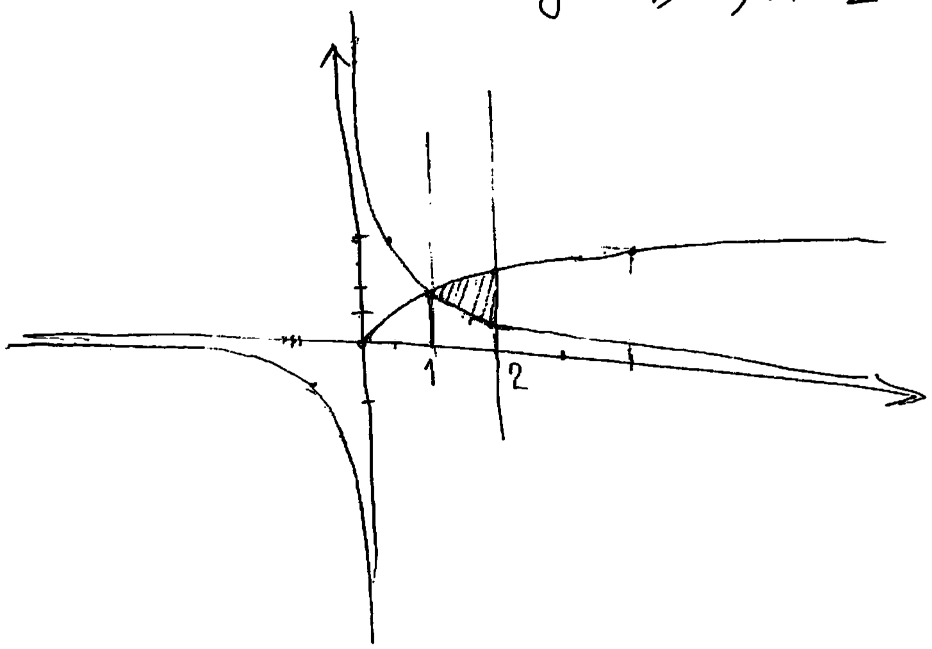
$$+\frac{3}{2}(1+3)=-\frac{244}{10}-\frac{80}{4}+\frac{28}{3}-\frac{8}{2}+\frac{12}{2}$$

$$\boxed{I=}\ \frac{2928-2400+1120-480+720}{120}=\frac{1888}{120}=\frac{472}{30}=\underline{\underline{\frac{236}{15}}}$$

---

24. $\iint_D y\ln x\,dxdy$, D ограничена линијама $xy=1$, $y=\sqrt{x}$, $x=2$

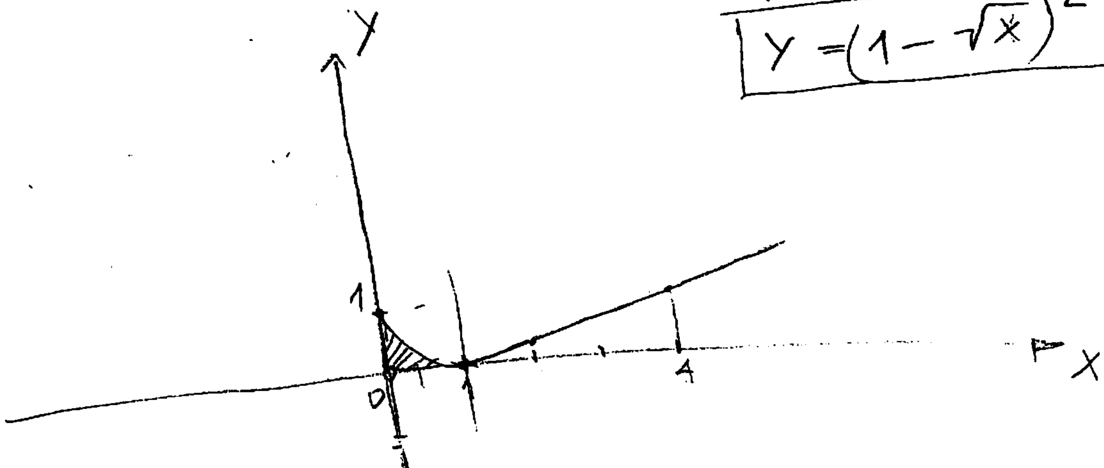
$y=\frac{1}{x}$



(28) $\iint\limits_D xy\,dx\,dy$, D ограничена координатним осама и параболом $\sqrt{x}+\sqrt{y}=1$

$\sqrt{y} = 1-\sqrt{x}$

$\boxed{y=(1-\sqrt{x})^2}$



$$I=\int_0^1 x\,dx\int_0^{(1-\sqrt{x})^2} y\,dy=\int_0^1 x\,dx\cdot\frac{y^2}{2}\Big|_0^{(1-\sqrt{x})^2}=\frac{1}{2}\int_0^1 x(1-\sqrt{x})^4\,dx$$

$$(1-\sqrt{x})(1-\sqrt{x})=(1-2\sqrt{x}+x)(1-2\sqrt{x}+x)=1-2\sqrt{x}+x$$
$$-2\sqrt{x}+4x-2\sqrt{x^3}+$$
$$+x-2\sqrt{x^3}+x^2$$
$$1-4\sqrt{x}+6x-4\sqrt{x^3}+x^2$$

$$\frac{x^{\frac{3}{2}+1=\frac{5}{2}}}{\frac{3}{2}+1=\frac{5}{2}}\qquad \frac{x^{\frac{5}{2}+1=\frac{7}{2}}}{\frac{7}{2}}$$

$$I=\frac{1}{2}\int_0^1\left(x-4\sqrt{x^3}+6x^2-4\sqrt{x^5}+x^3\right)dx=$$

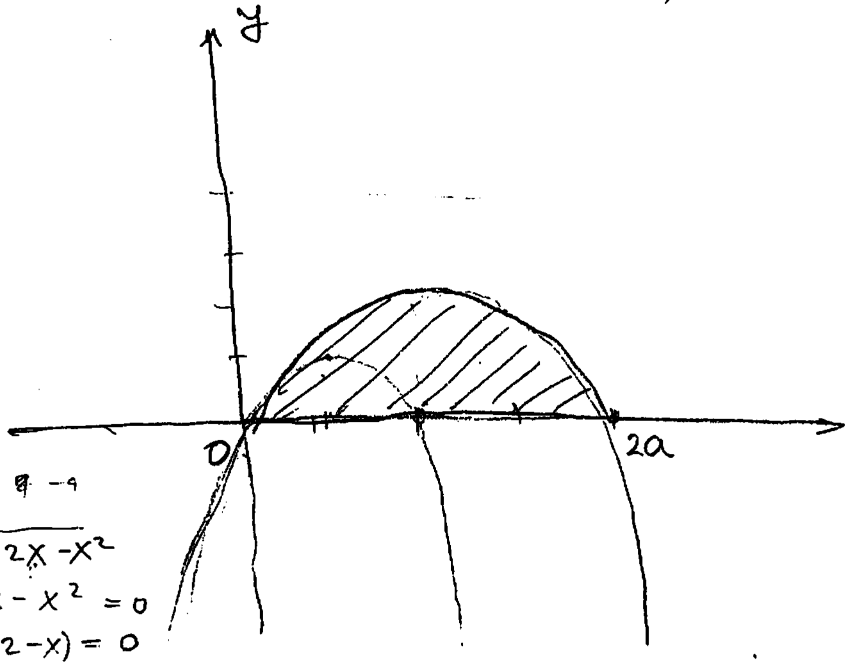
$$=\frac{1}{2}\left(\frac{x^2}{2}\Big|_0^1-4\cdot\frac{2}{5}x^{5/2}\Big|_0^1+\cancel{6}^{2}\frac{x^3}{\cancel{3}}\Big|_0^1-4\cdot\frac{2}{7}x^{7/2}\Big|_0^1+\frac{x^4}{4}\Big|_0^1\right)=$$

$$=\frac{1}{4}(1-0)-\frac{4}{5}(1-0)+(1-0)-\frac{4}{7}(1-0)+\frac{1}{8}(1-0)=$$

$$=\frac{1}{4}-\frac{4}{5}+1-\frac{4}{7}+\frac{1}{8}=\frac{70-244+280-160+35}{280}=\boxed{\frac{-19}{280}}$$

32. $\iint\limits_D x^2 y \, dx\, dy$, D ограничена линијама $y=0$

$y=\sqrt{2ax-x^2}$ $\quad a\neq 0$



$\cancel{a<0}$
$a=1 \longrightarrow y=\sqrt{2x-x^2}$
$a=2$

$2x-x^2=0$

$x(2-x)=0$

$\boxed{x=0 \wedge 2=x}$

$\longrightarrow y=\sqrt{4x-x^2} \rightarrow x_1=0 \wedge x_2=4$

$8-4$

$\underline{x_1=0 \wedge x_2=2a}$

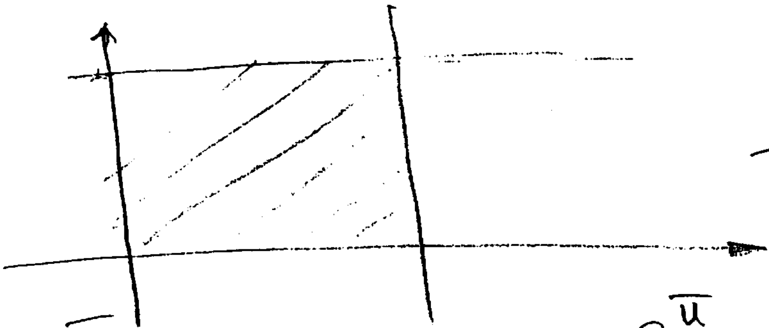
$\frac{y^2}{2}$

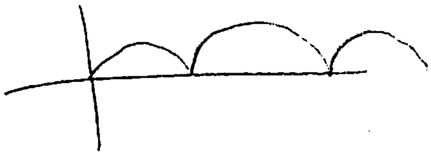
$$I=\int_0^{2a} x^2 dx \int_0^{\sqrt{2ax-x^2}} y\, dy = \int_0^{2a} x^2 dx \cdot \frac{1}{2}(2ax-x^2)=$$

$$=\frac{1}{2}\int_0^{2a}(2ax^3-x^4)\,dx = a\int_0^{2a} x^3 dx - \frac{1}{2}\int_0^{2a} x^4 dx =$$

$= \ldots\ldots$

(37) $\iint\limits_D |\cos(x+y)|\,dx\,dy \quad D=\{(x,y)\,|\,0\le x\le \pi,\ 0\le y\le \pi\}$





$$I=\int_0^{\pi} dx \int_0^{\pi} |\cos(x+y)|\,dy = \pm \int_0^{\pi} dx \int_0^{\pi} |\cos(x+y)|\,dy$$

$$I= \int_0^{\pi} dx \int_0^{\pi} |(\cos x \cos y - \sin x \sin y)|\,dy =$$

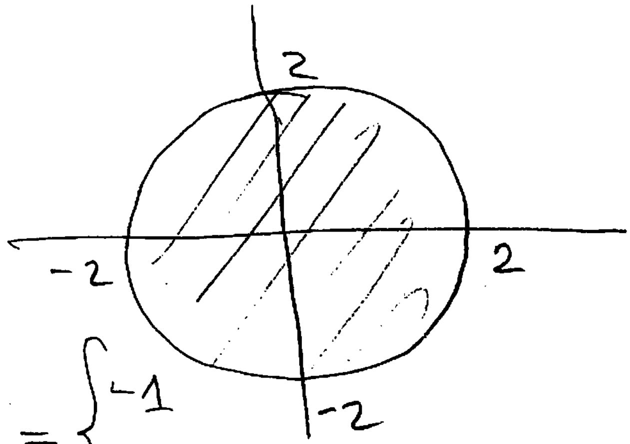
$$= \int_0^{\pi} dx \left[\int_0^{\pi} \cos x \cos y\,dy - \int_0^{\pi} \sin x \sin y\,dy\right] =$$

$$= \int_0^{\pi} dx \left[\cancel{-\cos x \sin y\Big|_0^{\pi}} - \sin x \cos y\Big|_0^{\pi}\right] =$$

$$= \int_0^{\pi} dx \left(-\sin x \left(\overset{-1}{\cancel{\cos \pi}} - \overset{1}{\cancel{\cos 0}}\right)\right) = 2\int_0^{\pi} \sin x\,dx =$$

$$= -2 \cos x\Big|_0^{\pi} = -2(\cos \pi - \cos 0) = \underline{\underline{4}}$$

38 $\iint_D \text{sgn}(x^2-y^2+2)\,dx\,dy$; $D=\{(x,y) \mid x^2+y^2 \leq 4\}$



$$\text{sgn}(x^2-y^2+2)=\begin{cases} -1 \\ , \end{cases}$$

$x^2-y^2+2=0$

$x^2-y^2=-2$

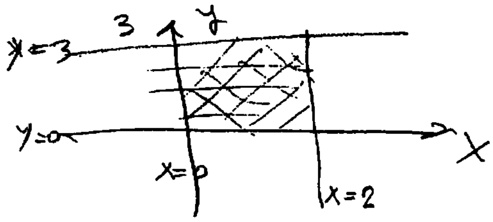
$x=0 \quad y=\pm\sqrt{2}$?

$y=0 \quad x=\pm i\sqrt{2}$

6.3.a)

(1.) $\int_0^2 dx \int_0^3 (x^2+2xy)\,dy =$

$$\begin{array}{l} 0 \le x \le 2 \\ 0 \le y \le 3 \\ \hline 0 \le y \le 3 \\ D: \; 0 \le x \le 2 \\ \hline \end{array}$$

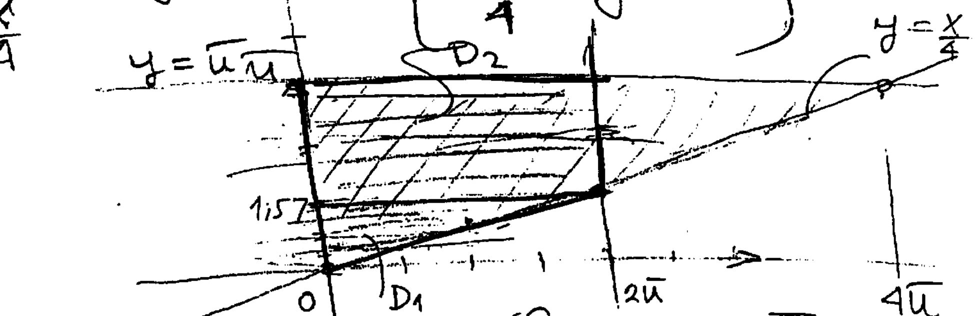


$\int_0^3 dy \int_0^2 (x^2+2xy)\,\cancel{dy}\,dx = \ldots\ldots$ ✓

$\frac{x}{4} = \pi$

$x = 4\pi$

$\int_0^{2\pi} \cos^2 x\,dx \int_{\frac{x}{4}}^{\pi} y\,dy \to D: \begin{cases} 0 \le x \le 2\pi \\ \frac{x}{4} \le y \le \pi \end{cases}$



$D_1: \begin{cases} 0 \le y \le \frac{\pi}{2} \\ 0 \le x \le 4y \end{cases}$

$D_2: \begin{cases} \frac{\pi}{2} \le y \le \pi \\ 0 \le x \le 2\pi \end{cases}$

$$I = \int_0^{\pi/2} y\,dy \int_0^{4y} \cos^2 x\,dx + \int_{\frac{\pi}{2}}^{\pi} y\,dy \int_0^{2\pi} \cos^2 x\,dx =$$

$$I=\int_0^{\pi/2} y\,dy\left[\frac{\cos x\sin x}{2}\Big|_0^{4y}+\frac{1}{2}\int_0^{4y}\cos x\,dx\right]+\quad \frac{\sin 2x}{4}$$

$$+\int_{\pi/2}^{\pi} y\,dy\left[\frac{\cos x\sin x}{2}\Big|_0^{2\pi}+\frac{1}{2}\int_0^{2\pi}\cos x\,dx\right]=$$
$$\frac{1}{2}(\cancel{\sin 2\pi}-\cancel{\sin 0})$$

$$=\int_0^{\pi/2} y\,dy\left[\frac{1}{2}(\cos 4y\sin 4y)+\frac{1}{2}(\sin 4y)\right]=$$

$$=\int_0^{\pi/2}\left(\frac{1}{4}\sin 8y+\frac{1}{2}\sin 4y\right)y\,dy=$$

$$=\frac{1}{4}\int_0^{\pi/2} y\sin 8y\,dy+\frac{1}{2}\int_0^{\pi/2} y\sin 4y\,dy=$$

$$\left\{\begin{array}{l} u=y\Rightarrow du=dy \\ v=-\frac{1}{8}\cos 8y \end{array}\right\}$$

$$=\frac{1}{4}\left(-\frac{1}{8}y\cos 8y\Big|_0^{\pi/2}+\frac{1}{8}\int_0^{\pi/2}\cos 8y\,dy\right)+\frac{1}{2}\left(-\frac{1}{4}y\cos 4y\Big|_0^{\pi/2}+\right.$$

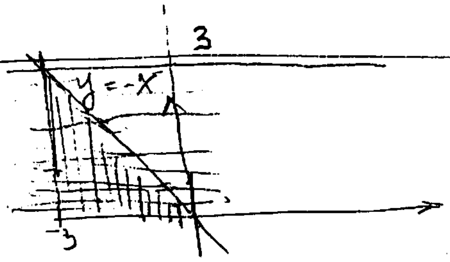
$$\left.+\frac{1}{4}\int_0^{\pi/2}\cos 4y\,dy\right)=$$

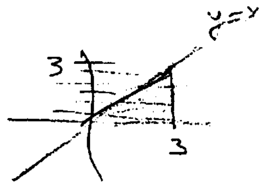
$$=-\frac{1}{32}\,\frac{\pi}{2}\cancel{\cos 4\pi}^{\,1}+\frac{1}{64}(\cancel{\sin 4\pi}^{\,0}-\cancel{\sin 0}^{\,0})-\frac{1}{8}\,\frac{\pi}{2}\cancel{\cos 2\pi}_{\,1}+$$

$$+\frac{1}{16}(\cancel{\sin 2\pi}^{\,0}-\cancel{\sin 0}_{\,0})=-\frac{\pi}{64}-\frac{\pi}{16}=\boxed{-\frac{5\pi}{64}}$$

(9) $\int_{-3}^{0} dx \int_{-x}^{3} e^{-y^2} dy + \int_{0}^{3} dx \int_{x}^{3} e^{-y^2} dy$

$D: \begin{cases} 0 \le x \le -3 \\ -x \le y \le 3 \end{cases}$ $\qquad D: \begin{cases} 0 \le x \le 3 \\ x \le y \le 3 \end{cases}$





$D: \begin{cases} 0 \le y \le 3 \\ -y \le x \le -3 \end{cases}$ $\qquad D: \begin{cases} 0 \le y \le 3 \\ y \le x \le 3 \end{cases}$

$\int_0^3 e^{-y^2} dy \int_{-y}^{-3} dx + \int_0^3 e^{-y^2} dy \int_y^3 dx =$

$-\int_0^3 e^{-y^2} dy \, (-3+y) + \int_0^3 e^{-y^2} dy \, (3-y) =$

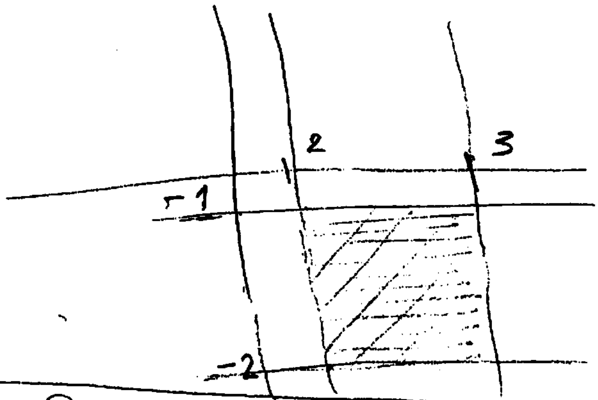
$= -3\int_0^3 e^{-y^2} dy + \int_0^3 y e^{-y^2} dy + 3\int_0^3 e^{-y^2} dy - \int_0^3 y e^{-y^2} dy =$

?

④ $\int_{-2}^{3} dx \int_{-2}^{-1} f(x,y)\,dy = \left\{ \begin{array}{c} 2 \le x \le 3 \\ -2 \le y \le -1 \end{array} \right\} =$

$-2 \le y \le -1$

$2 \le x \le 3$



$I = \int_{2}^{3} dy \int_{-2}^{-1} f(x,y)\,dx$

⑤ $\int_{0}^{1} dx \int_{0}^{2-2x} f(x,y)\,dy = \left\{ \begin{array}{c} 0 \le x \le 1 \\ 0 \le y \le 2-2x \end{array} \right\} \Rightarrow$

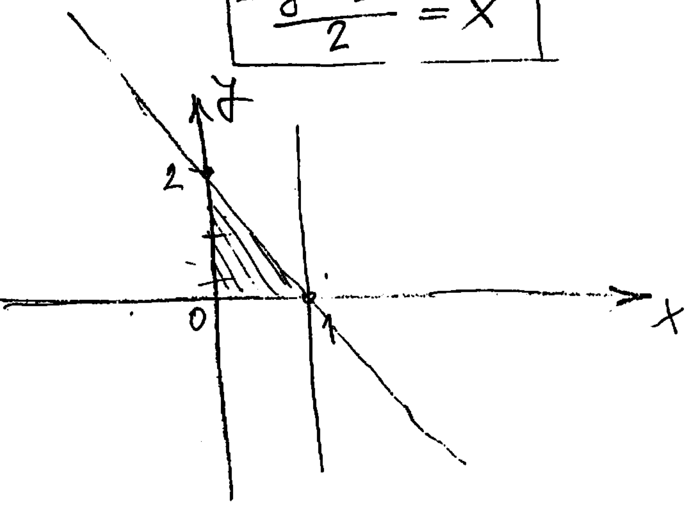
$\Rightarrow \left\{ \begin{array}{c} 0 \le y \le 2 \\ 0 \le x \le \frac{-y+2}{2} \end{array} \right\}$

$y = 2-2x$

$y-2 = -2x$

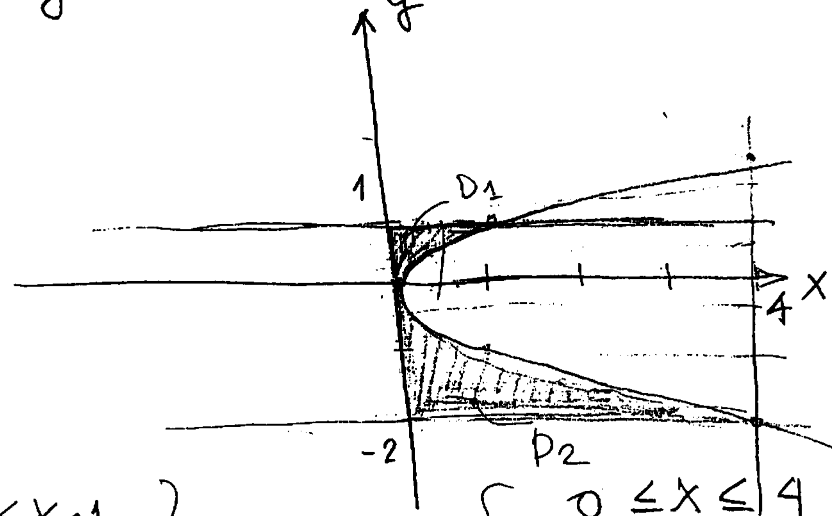
$\boxed{\frac{-y+2}{2} = x}$

$= \int_{0}^{2} dy \int_{0}^{\frac{-y+2}{2}} f(x,y)\,dx$



G.D.L

(20.) $\displaystyle\int_{-2}^{1} dy \int_{y^2}^{4} f(x,y)\,dx = \left\{ \begin{array}{l} -2 \le \cancel{x} \le 1 \\ y^2 \le x \le 4 \end{array} \right\}$



$y^2 \le x$

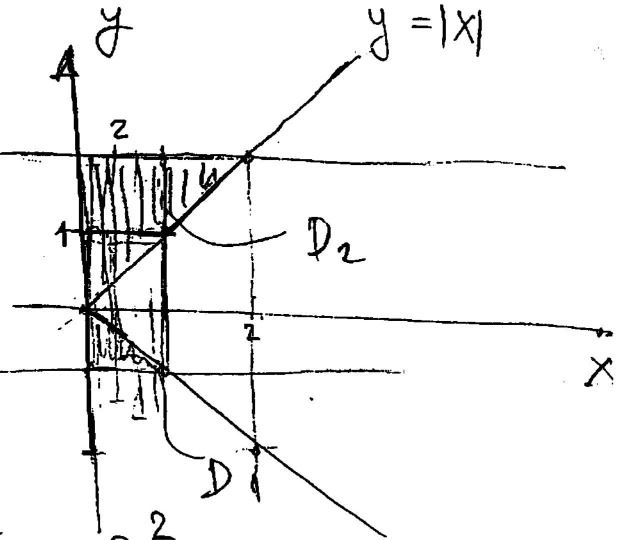
$x = 1$

$y = \sqrt{x}$

$D_1 : \left\{ \begin{array}{l} 0 \le x \le 1 \\ \sqrt{x} \le y \le 1 \end{array} \right\}$ $\quad D_2 : \left\{ \begin{array}{l} 0 \le x \le 4 \\ -2 \le y \le -\sqrt{x} \end{array} \right\}$

$$= \int_0^1 dx \int_{\sqrt{x}}^{1} f(x,y)\,dy + \int_0^4 dx \int_{-2}^{-\sqrt{x}} f(x,y)\,dy$$

8. $$\int_{-1}^{2} dy \int_{0}^{|y|} f(x,y)\,dx = \left\{ \begin{array}{l} -1 \le y \le 2 \\ 0 \le x \le |y| \end{array} \right\} =$$



$D_1: \left\{ \begin{array}{l} 0 \le x \le 1 \\ -1 \le y \le -x \end{array} \right\}$

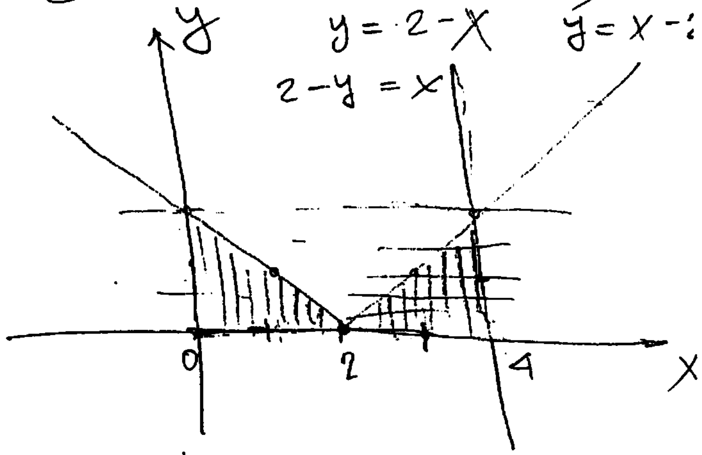
$D_2: \left\{ \begin{array}{l} 0 \le x \le 2 \\ x \le y \le 2 \end{array} \right\}$

$$= \int_{0}^{1} dx \int_{-1}^{-x} f(x,y)\,dy + \int_{0}^{2} dx \int_{|x|}^{2} f(x,y)\,dy$$

13. $$\int_{0}^{4} dx \int_{0}^{|x-2|} f(x,y)\,dy = \left\{ \begin{array}{l} 0 \le x \le 4 \\ 0 \le y \le |x-2| \end{array} \right\}$$

$D_1: \left\{ \begin{array}{l} 0 \le y \le 2 \\ 0 \le x \le 2-y \end{array} \right\}$

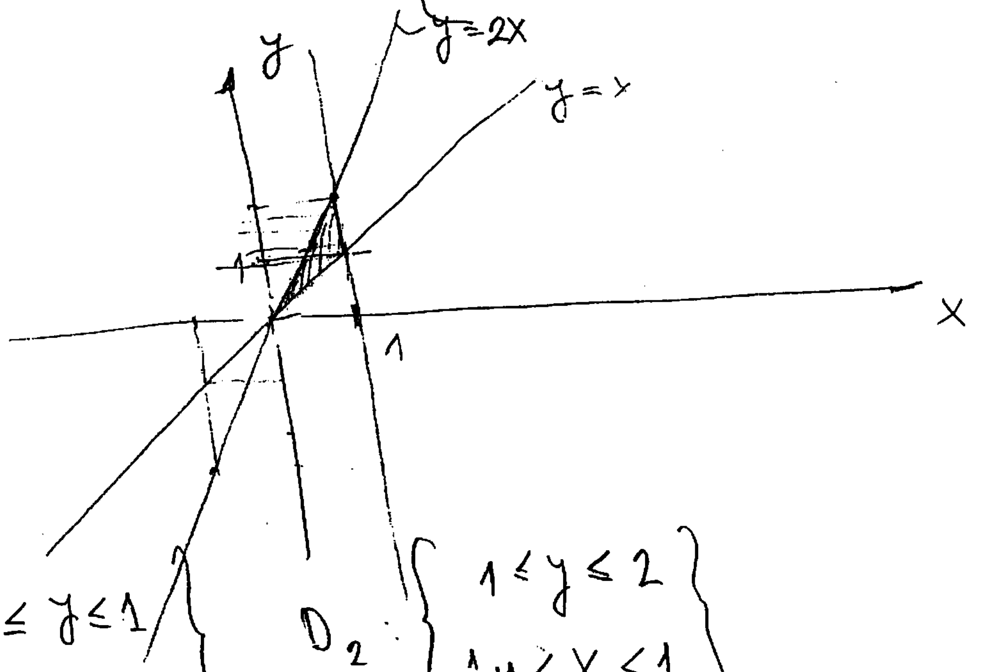
$D_2: \left\{ \begin{array}{l} 0 \le y \le 2 \\ y+2 \le x \le 4 \end{array} \right\}$



$$= \int_{0}^{2} dy \int_{0}^{2-y} f(x,y)\,dx + \int_{0}^{2} dy \int_{y+2}^{4} f(x,y)\,dx$$

6.3. 4)

3.) $\int_0^1 dx \int_x^{2x} (x-y+1)\,dy = \left\{ D: \left[\begin{matrix} 0 \le x \le 1 \\ x \le y \le 2x \end{matrix} \right] \right\}$



$D_1: \left\{ \begin{matrix} 0 \le y \le 1 \\ \frac{1}{2}y \le x \le y \end{matrix} \right\}$ $\quad D_2: \left\{ \begin{matrix} 1 \le y \le 2 \\ \frac{1}{2}y \le x \le 1 \end{matrix} \right\}$

$$= \int_0^1 dy \int_{\frac{1}{2}y}^{y} (x-y+1)\,dx + \int_1^2 dy \int_{\frac{1}{2}y}^{1} (x-y+1)\,dx =$$

$$= \int_0^1 dy \left(\frac{x^2}{2}\Big|_{\frac{1}{2}y}^{y} - xy\Big|_{\frac{1}{2}y}^{y} + x\Big|_{\frac{1}{2}y}^{y} \right) + \int_1^2 dy \left(\frac{x^2}{2}\Big|_{\frac{1}{2}y}^{1} - yx\Big|_{\frac{1}{2}y}^{1} + x\Big|_{\frac{1}{2}y}^{1} \right.$$

$$= \int_0^1 \left[\frac{1}{2}\left(y^2 - \frac{1}{4}y^2\right) - y\left(y - \frac{1}{2}y\right) + \left(y - \frac{1}{2}y\right) \right] dy +$$

$$+ \int_1^2 \left[\frac{1}{2}\left(1 - \frac{1}{4}y^2\right) - y\left(1 - \frac{1}{2}y\right) + \left(1 - \frac{1}{2}y\right) \right] dy \;\underset{\frac{-1+4}{8}}{=}$$

$$= \int_0^1 \left(\frac{3}{8}y^2 - \frac{1}{2}y^2 + \frac{1}{2}y \right) dy + \int_1^2 \left(\frac{1}{2} - \frac{1}{8}y^2 - y + \frac{1}{2}y^2 + 1 - \frac{1}{2}y \right) dy =$$

$\frac{3-4}{8} = -\frac{1}{8}$ $\qquad -\frac{2}{2} - \frac{1}{2} = -\frac{3}{2}$

$$=\int_0^1\left(-\frac{1}{8}y^2+\frac{1}{2}y\right)dy+\int_1^2\left(\frac{3}{8}y^2-\frac{3}{2}y+\frac{3}{2}\right)dy=$$

$12\cdot3=$

$$=-\frac{1}{8}\frac{y^3}{3}\Big|_0^1+\frac{1}{2}\frac{y^2}{2}\Big|_0^1+\frac{3}{8}\frac{y^3}{3}\Big|_1^2-\frac{3}{2}\frac{y^2}{2}\Big|_1^2+\frac{3}{2}y\Big|_1^2=$$

$$=-\frac{1}{24}+\frac{1}{4}+\frac{1}{8}(8-1)-\frac{3}{4}(4-1)+\frac{3}{2}=$$

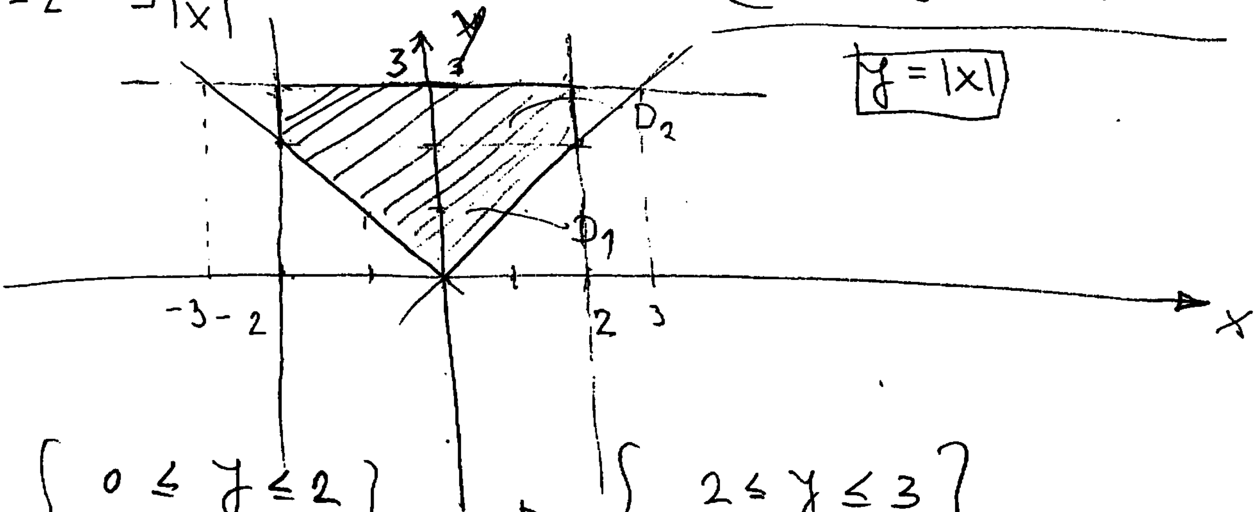
$$=-\frac{1}{24}+\frac{1}{4}+\frac{7}{8}-\frac{9}{4}+\frac{3}{2}=\frac{-1+6+21-54+36}{24}=\frac{8}{24}$$

$$=\boxed{\frac{1}{3}}$$

---

(8) $$\int_{-2}^{2}dx\int_{|x|}^{3}(xy-y^2)\,dy=\left\{\begin{array}{l}-2\le x\le 2\\ |x|\le y\le 3\end{array}\right\}$$

$\boxed{y=|x|}$



$$D_1:\left\{\begin{array}{l}0\le y\le 2\\ -y\le x\le y\end{array}\right\}\qquad D_2:\left\{\begin{array}{l}2\le y\le 3\\ -2\le x\le 2\end{array}\right\}$$

$$=\int_0^2dy\int_{-y}^{y}(xy-y^2)\,dx+\int_2^3dy\int_{-2}^{2}(xy-y^2)\,dy=$$

$$=\int_0^2dy\left(y\frac{x^2}{2}\Big|_{-y}^{y}-y^2x\Big|_{-y}^{y}\right)+\int_2^3dy\left(y\frac{x^2}{2}\Big|_{-2}^{2}-y^2x\Big|_{-2}^{2}\right)=$$

$$= \int_0^2 \left(\cancel{y\left(\frac{y^3}{y^2} - \frac{y^2}{2}\right)}^{\,0} - y^2\left(\overset{2y}{y+y}\right)\right) dy \; +$$

$$+ \int_2^3 \left(\cancel{y(2-2)}^{\,0} - y^2(2+2)\right) dy =$$

2 · 2 · 2 · 2

2 · 3 · 3

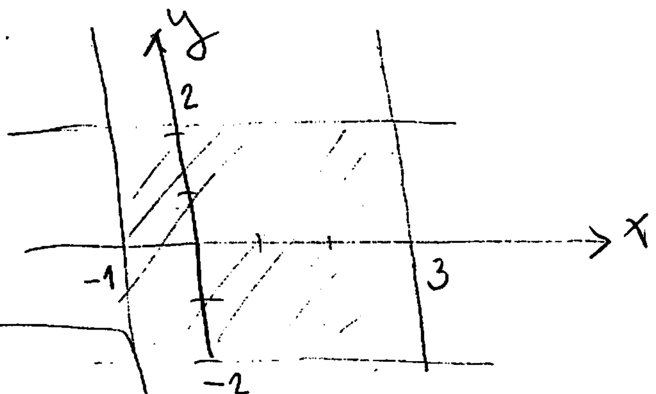
$$= -2\int_0^2 y^3\,dy - 4\int_2^3 y^2\,dy = -2\cdot\frac{y^4}{4}\Big|_0^2 - 4\,\frac{y^3}{3}\Big|_2^3 =$$

$$= -8 - \frac{4}{3}\left(27 - 8\right) = -8 - \frac{76}{3} = \frac{-24-76}{3} =$$

$$= \frac{100}{3} = \boxed{\frac{100}{3}}$$

B) 3) $\int_{-1}^{3} dx \int_{-2}^{2} f(x,y)\,dy = \begin{cases} -1 \le x \le 3 \\ -2 \le y \le 2 \end{cases}$

$$D: \begin{cases} -2 \le y \le 2 \\ -1 \le x \le 3 \end{cases}$$



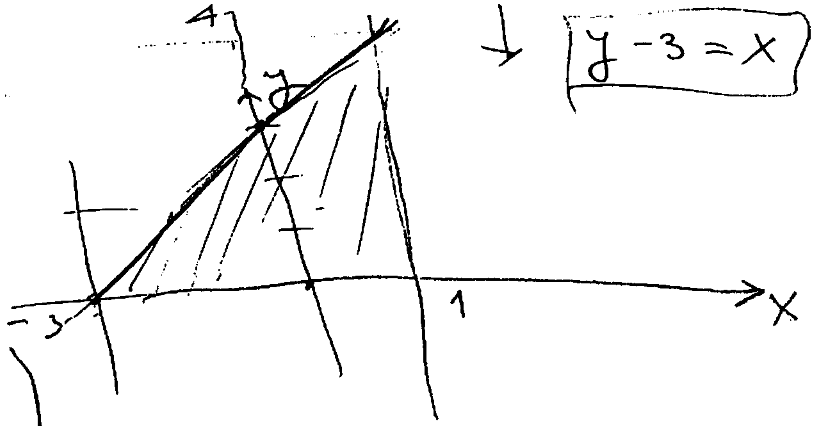
$$\boxed{= \int_{-2}^{2} dy \int_{-1}^{3} f(x,y)\,dx}$$

⑥ $\displaystyle\int_{-3}^{1} dx \int_{0}^{x+3} f(x,y)\,dy$

$-3 \le x \le 1$

$0 \le y \le x+3$

$\downarrow$ $\boxed{y-3=x}$

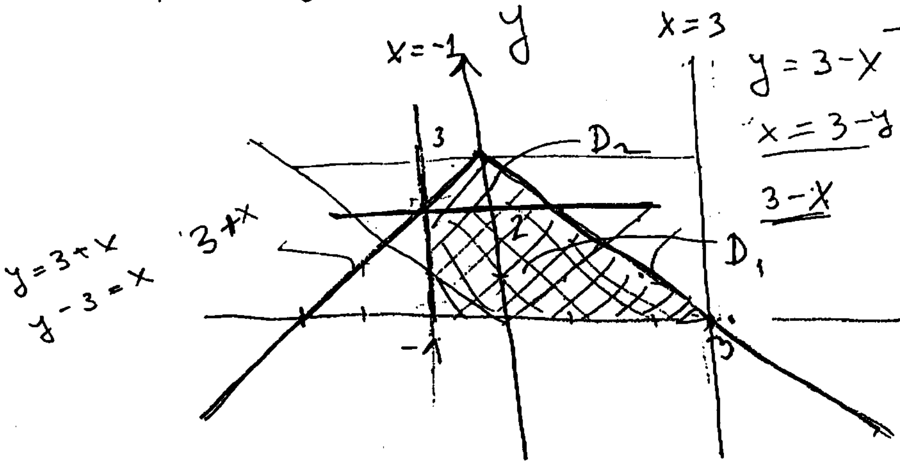


$$\boxed{= \int_{0}^{4} dy \int_{y-3}^{1} f(x,y)\,dx}$$

⑭ $\displaystyle\int_{-1}^{3} dx \int_{0}^{3-|x|} f(x,y)\,dy$

$-1 \le x \le 3$

$0 \le y \le 3-|x|$



$D_1: \begin{cases} 0 \le y \le 2 \\ -1 \le x \le 3-y \end{cases}$

$D_2: \begin{cases} 2 \le y \le 3 \\ y-3 \le x \le 3-y \end{cases}$

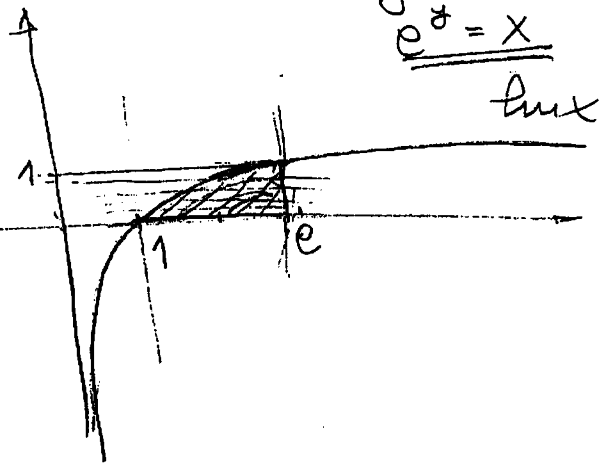
$$\boxed{= \int_{0}^{2} dy \int_{-1}^{3-y} f(x,y)\,dx + \int_{2}^{3} dy \int_{y-3}^{3-y} f(x,y)\,dx}$$

(21) $\int_1^e dx \int_0^{\ln x} f(x,y)\,dy = \left\{ \begin{array}{l} 1 \le x \le e \\ 0 \le y \le \ln x \end{array} \right\}$

$y = \ln x$

$\underline{\underline{e^y = x}}$

$= \left\{ \begin{array}{l} 0 \le y \le 1 \\ e^y \le x \le e \end{array} \right\}$



$= \int_0^1 dy \int_{e^y}^{e} f(x,y)\,dx$

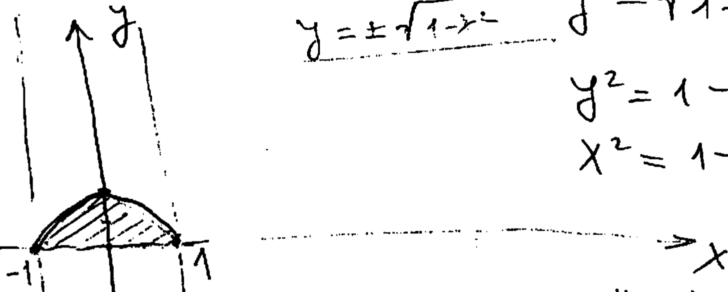
(26.) $\int_{-1}^{1} dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y)\,dy = \left\{ \begin{array}{l} -1 \le x \le 1 \\ -\sqrt{1-x^2} \le y \le \sqrt{1-x^2} \end{array} \right\}$

$y = \pm\sqrt{1-x^2}$ $\quad y = \sqrt{1-x^2}$

$y^2 = 1 - x^2$

$x^2 = 1 - y^2$

$\left\{ \begin{array}{l} 0 \le y \le 1 \\ -\sqrt{1-y^2} \le x \le \sqrt{1-y^2} \end{array} \right\}$



$\underline{x = \pm\sqrt{1-y^2}}$

$= \int_0^1 dy \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x,y)\,dx$

29. $\displaystyle\int_{-7}^{1} dy \int_{2-\sqrt{7-6y-y^2}}^{2+\sqrt{7-6y-y^2}} f(x,y)\,dx$

$$\left\{\begin{array}{l} -7 \le y \le 1 \\ 2-\sqrt{\dots} \le x \le 2+\sqrt{\dots} \end{array}\right\}$$

$$\boxed{\begin{array}{l} x = 2+\sqrt{7-6y-y^2} \\ x = 2-\sqrt{7-6y-y^2} \end{array}}$$

$$\boxed{(x-2)^2+(y+3)^2=16}$$

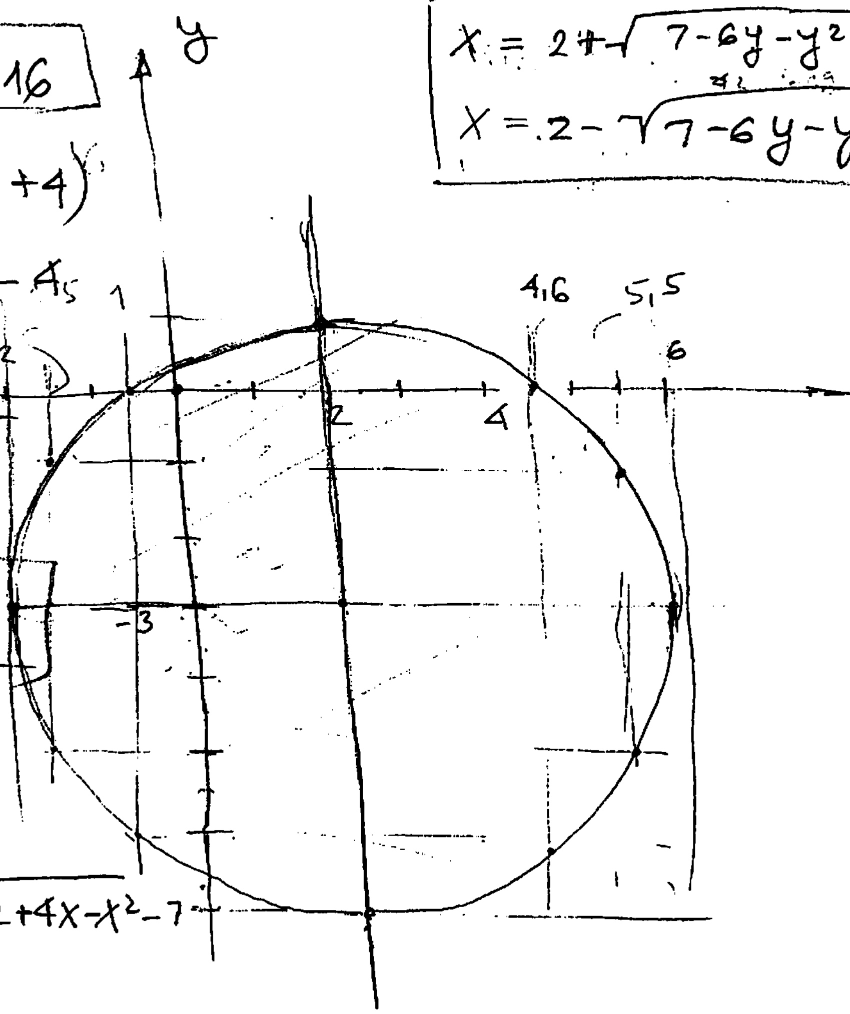
$(y+3)^2 = 16-(x^2-4x+4)$

$(y+3)^2 = 16-x^2+4x-4$

$(y+3)^2 = -x^2+4x+12$

$y+3 = \pm\sqrt{12+4x-x^2}$

$$\boxed{y = -3 \pm \sqrt{12+4x-x^2}}$$



$$= \int_{-2}^{6} dx \int_{-3-\sqrt{12+4x-x^2}}^{-3+\sqrt{12+4x-x^2}} f(x,y)\,dy$$

(33.) $$\int_0^{1/2} dx \int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} f(x,y)\,dy \Rightarrow \begin{array}{l} 0 \le x \le \frac{1}{2} \\ 1-\sqrt{1-x^2} \le y \le 1+\sqrt{1-x^2} \end{array}$$

$$y = 1 \pm \sqrt{1-x^2}$$

$$(y-1)^2 = 1-x^2$$

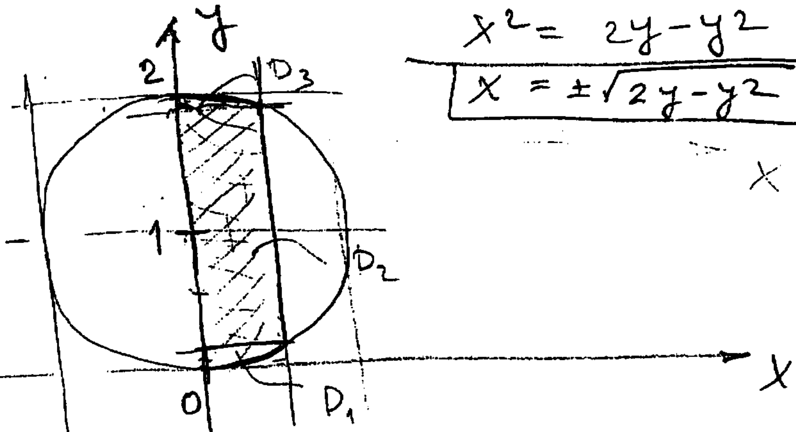
$$x^2 + (y-1)^2 = 1$$

$$x^2 = 1-(y^2-2y+1)$$

$$x^2 = 1-y^2+2y-1)$$

$$x^2 = 2y-y^2$$

$$\boxed{x = \pm\sqrt{2y-y^2}}$$



$$1-\frac{1}{4} = \frac{3}{4}$$

$$1 \pm \frac{\sqrt{3}}{2} = \begin{cases} 1{,}866 \\ 0{,}134 \end{cases}$$

$$= \int_0^{1-\frac{\sqrt{3}}{2}} dy \int_0^{-\sqrt{2y-y^2}} f(x,y)\,dx + \int_{1-\frac{\sqrt{3}}{2}}^{1+\frac{\sqrt{3}}{2}} dy \int_0^{1/2} f(x,y)\,dx +$$

$$+ \int_{1+\frac{\sqrt{3}}{2}}^{2} dy \int_0^{\sqrt{2y-y^2}} f(x,y)\,dx.$$

36.

$$\int_0^1 dx \int_{(1-x)^2/2}^{\sqrt{1-x^2}} f(x,y)\,dy =$$

$y^2 = 1-x^2$

$x^2+y^2 = \underline{1}$

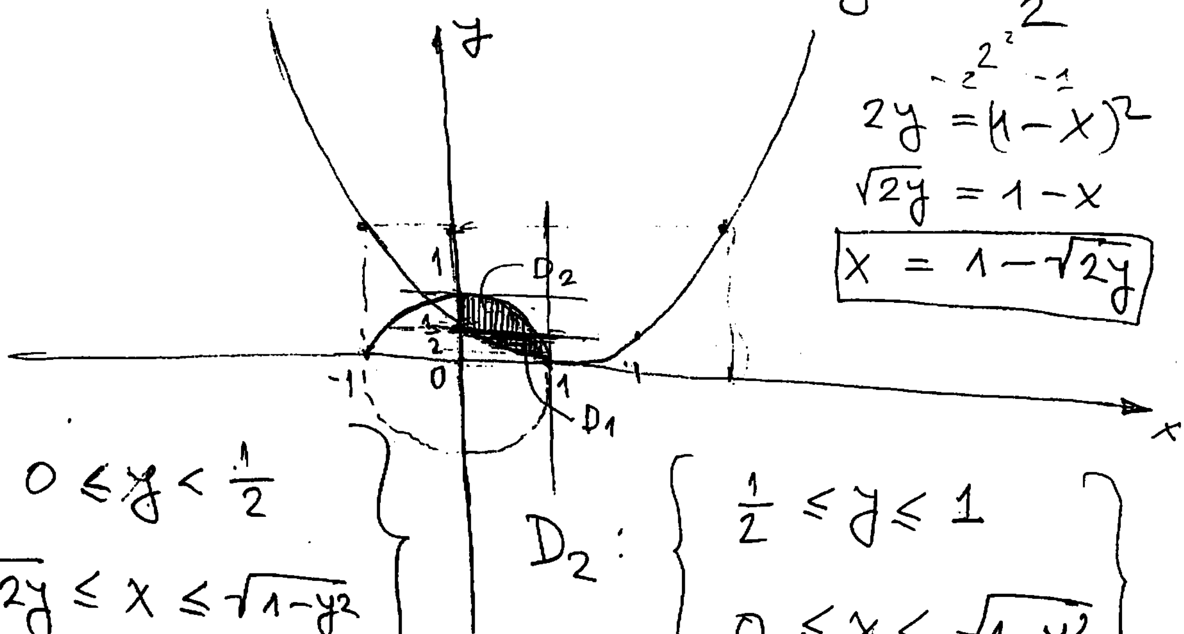
$x^2 = 1-y^2$

$0 \le x \le 1$

$\frac{(1-x)^2}{2} \le y \le \sqrt{1-x^2}$

$y = \sqrt{1-x^2}$

$y = \frac{(1-x)^2}{2}$



$2y = (1-x)^2$

$\sqrt{2y} = 1-x$

$\boxed{x = 1-\sqrt{2y}}$

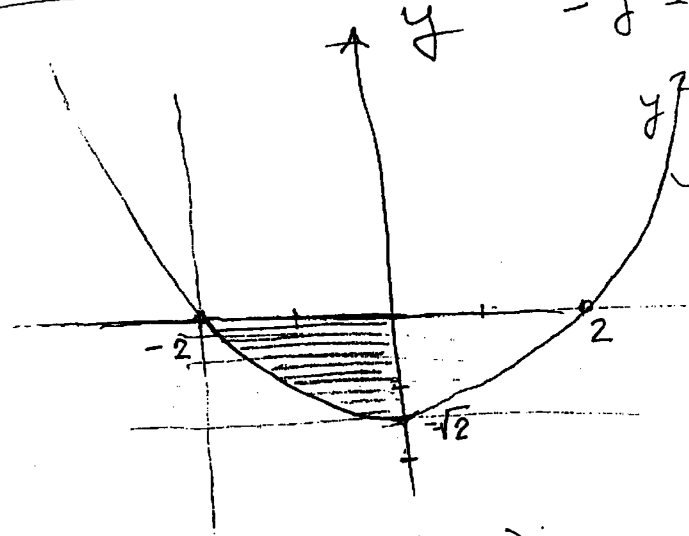
$D_1: \begin{cases} 0 \le y < \frac{1}{2} \\ 1-\sqrt{2y} \le x \le \sqrt{1-y^2} \end{cases}$ $D_2: \begin{cases} \frac{1}{2} \le y \le 1 \\ 0 \le x \le \sqrt{1-y^2} \end{cases}$

$$= \int_0^{\frac{1}{2}} dy \int_{1-\sqrt{2y}}^{\sqrt{1-y^2}} f(x,y)\,dx + \int_{\frac{1}{2}}^{1} dy \int_0^{\sqrt{1-y^2}} f(x,y)\,dx$$

(39.) $\int_{-2}^{0} dx \int_{-\sqrt{2-(x^2/2)}}^{0} f(x,y)\,dy =$

$-2 \leq x \leq 0$

$-\sqrt{2-(x^2/2)} \leq y \leq 0$



$y = -\sqrt{2-\left(\frac{x^2}{2}\right)}$

$-y = \sqrt{2-\frac{x^2}{2}}$

$y^2 = 2 - \frac{x^2}{2}$

$y^2 = \frac{4-x^2}{2}$

$2y^2 = 4 - x^2$

$2y^2 - 4 = -x^2$

$4 - 2y^2 = x^2$

$\boxed{-\sqrt{4-2y^2} = x}$

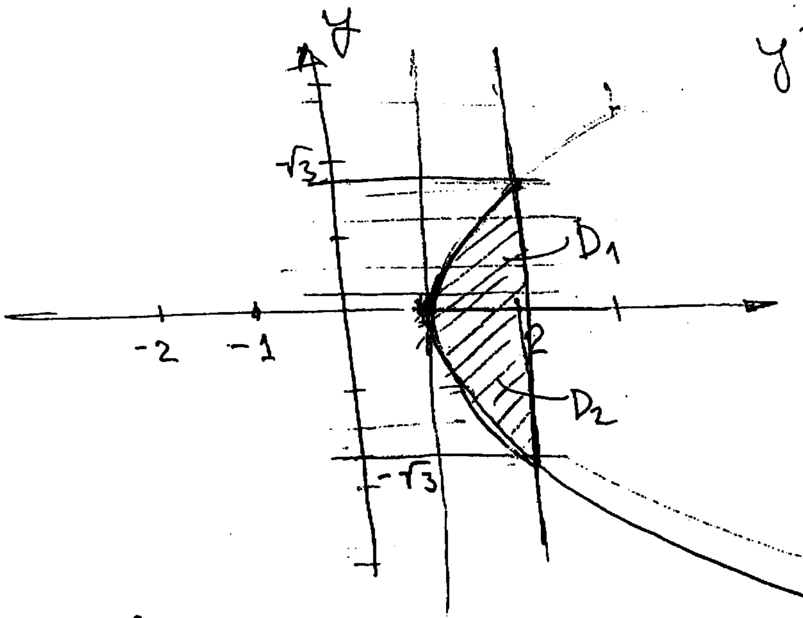
$$\left\{ \begin{array}{c} -\sqrt{2} \leq y \leq 0 \\ -\sqrt{4-2y^2} \leq x \leq 0 \end{array} \right\}$$

$$= \int_{-\sqrt{2}}^{0} dy \int_{-\sqrt{4-2y^2}}^{0} f(x,y)\,dx$$

(42.) $$\int_1^2 dx \int_{-\sqrt{x^2-1}}^{\sqrt{x^2-1}} f(x,y)\,dy =$$

$$\begin{cases} 1 \le x \le 2 \\ -\sqrt{x^2-1} \le y \le \sqrt{x^2-1} \end{cases}$$

$$y = \pm\sqrt{x^2-1}$$
$$y^2 = x^2-1$$
$$y^2 - x^2 = -1$$
$$-x^2 = -y^2-1$$
$$x^2 = y^2+1$$
$$\boxed{x = \pm\sqrt{y^2+1}}$$



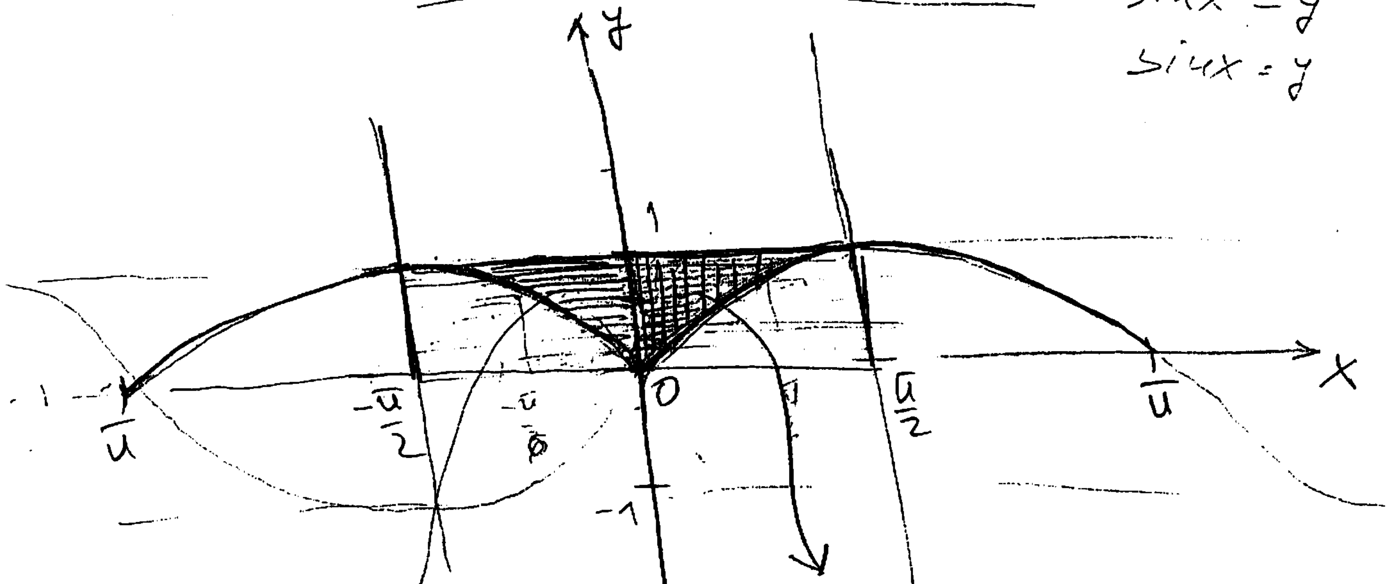
$$D_1: \begin{cases} 0 \le y \le \sqrt{3} \\ \sqrt{y^2+1} \le x \le 2 \end{cases} \qquad D_2: \begin{cases} -\sqrt{3} \le y \le 0 \\ \sqrt{y^2+1} \le x \le 2 \end{cases}$$

$$= \int_0^{\sqrt{3}} dy \int_{\sqrt{y^2+1}}^{2} f(x,y)\,dx + \int_{-\sqrt{3}}^{0} dy \int_{\sqrt{y^2+1}}^{2} f(x,y)\,dx$$

(46.) $$\int_{-\pi/2}^{\pi/2} dx \int_{|\sin x|}^{1} f(x,y)\,dy =$$

$$\begin{cases} -\frac{\pi}{2} \le x \le \frac{\pi}{2} \\ |\sin x| \le y \le 1 \end{cases}$$

$-\sin x = y$
$\sin x = y$



$$\begin{cases} 0 \le y \le 1 \\ \arcsin y \le x \le 0 \end{cases} \qquad \begin{cases} 0 \le y \le 1 \\ 0 \le x \le \arcsin y \end{cases}$$

$=$

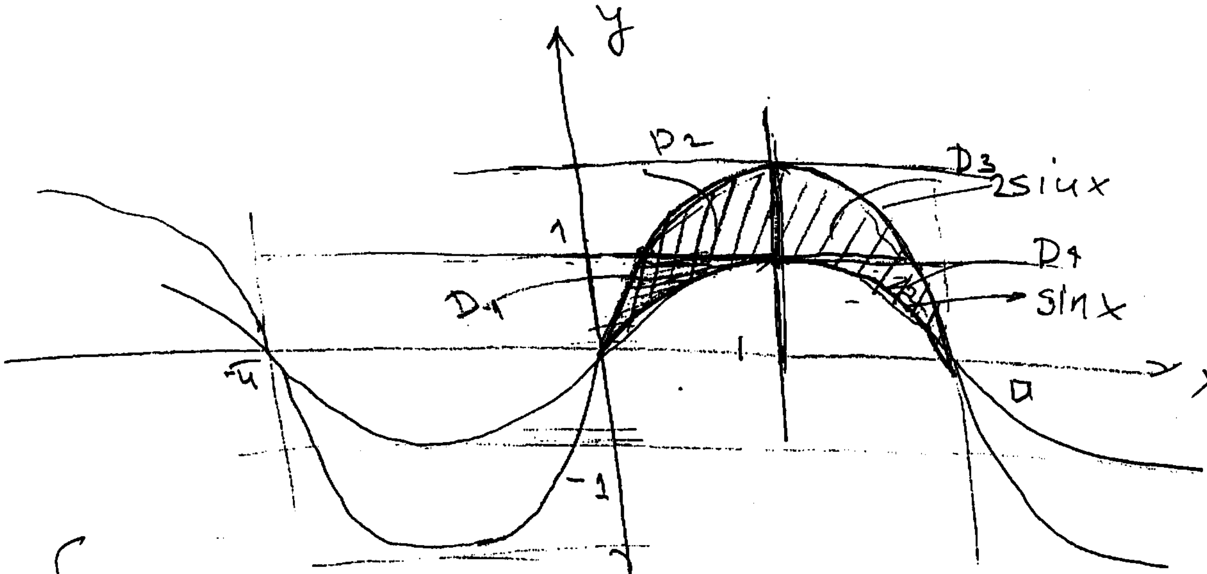
$$\begin{cases} 0 \le y \le 1 \\ -\arcsin y \le x \le \arcsin y \end{cases}$$

$$= \boxed{\int_0^1 dy \int_{-\arcsin y}^{\arcsin y} f(x,y)\,dx}$$

(47) $\int_0^{\pi} dx \int_{\sin x}^{2\sin x} f(x,y)\,dy =$

$0 \le x \le \pi$

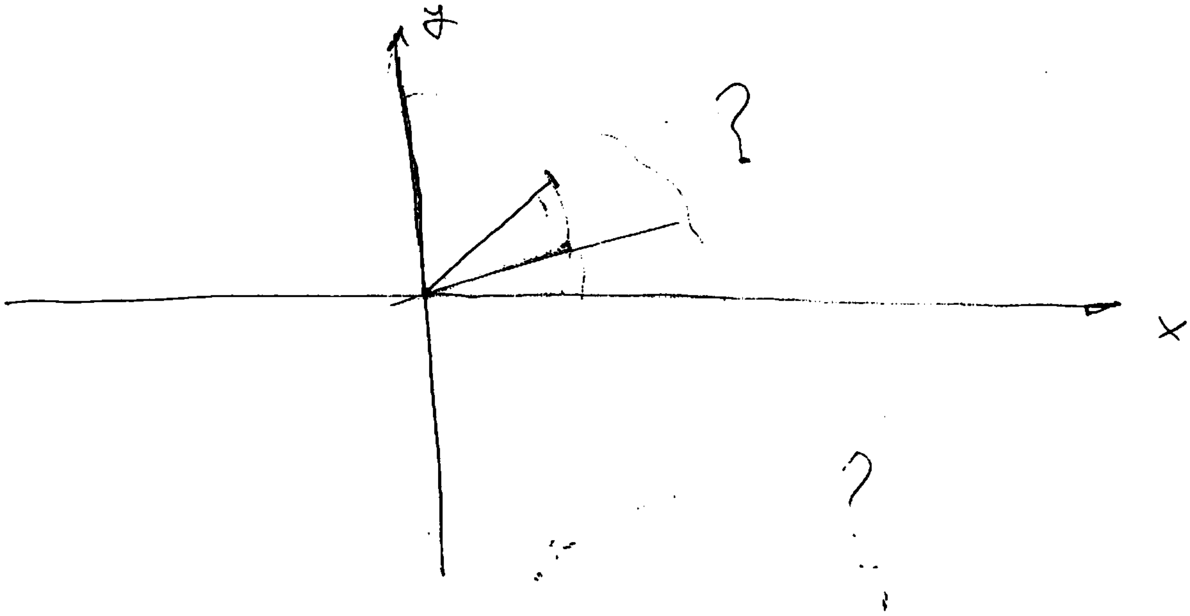
$\sin x \le y \le 2\sin x$



$D_1: \begin{cases} 0 \le y \le 1 \\ \arcsin 2y \le x \le \arcsin y \end{cases}$ $D_2: \begin{cases} 1 \le y \le 2 \\ \arcsin 2y \le x \le \frac{\pi}{2} \end{cases}$

$$= 2\cdot \int_0^1 dy \int_{\arcsin 2y}^{\arcsin y} f(x,y)\,dx + 2\cdot \int_1^2 dy \int_{\arcsin 2y}^{\frac{\pi}{2}} f(x,y)\,dx$$

52. $\int_0^{\pi/2} d\varphi \int_0^{\sqrt{\sin 2\varphi}} f(\rho, \varphi)\, d\rho$



$$\left\{ \begin{array}{l} 0 \le \varphi \le \pi/2 \\ 0 \le \rho \le \sqrt{\sin 2\varphi} \end{array} \right\}$$

$\frac{\pi}{4}$

$\rho^2 = \sin 2\varphi$

$\rho = 1 \rightarrow \frac{\pi}{4} = \varphi$

$\frac{\sqrt{3}}{2}$

55. $$\int_3^7 dy \int_{\frac{9}{y}}^{3} f(x,y)\,dx + \int_7^9 dy \int_{\frac{9}{y}}^{10-y} f(x,y)\,dx =$$

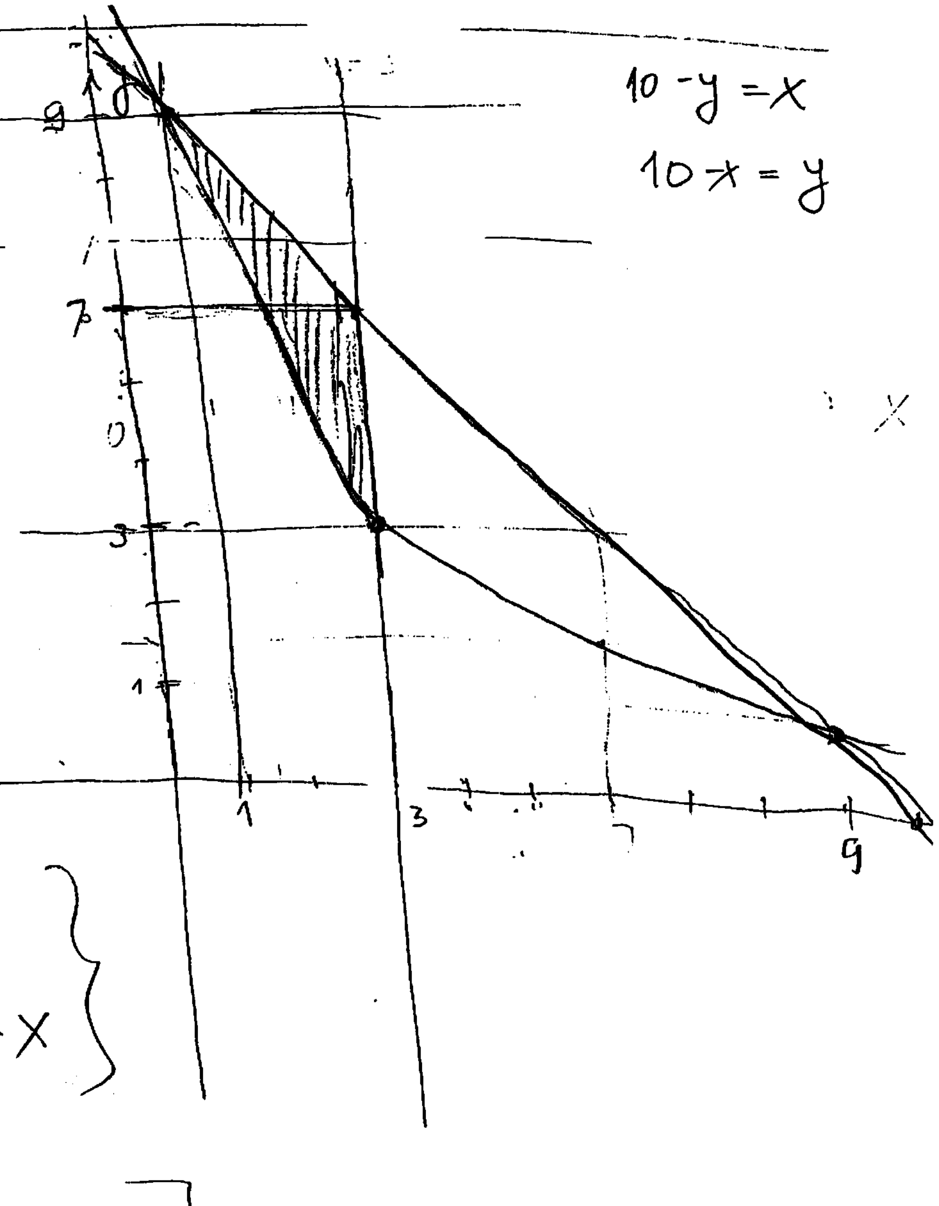
$3 \le y \le 7$ $\qquad$ $7 \le y \le 9$

$\frac{9}{y} \le x \le 3$ $\qquad$ $\frac{9}{y} \le x \le 10-y$

$\frac{9}{y} = x \Rightarrow \boxed{\frac{9}{x} = y}$

$10 - y = x$

$10 - x = y$



$$\begin{cases} 1 \le x \le 3 \\ \frac{9}{x} \le y \le 10-x \end{cases}$$

$$= \int_1^3 dx \int_{\frac{9}{x}}^{10-x} f(x,y)\,dy$$